The Mechanics of Fluids

The properties of fluids
A **fluid** does not maintain a definite shape but takes the shape of its container. Although gases and liquids share the property of being fluids, a sample of liquid has definite volume while a gaseous substance takes the volume as well as the shape of its container.

Density
The **density** of a substance of uniform composition is defined as its mass per unit volume:

$$\rho = \frac{m}{V}$$

The units of density in the SI system are kg/m³. The ratio of the density of a substance to the density of water at 4° C (1.0 X 10³ kg/m³) is called the **specific gravity** of the substance. The specific gravity of copper at STP is 9. This means that a certain volume of copper would be nine times heavier than an equal volume of water.

Pressure
An object submerged in a fluid will experience a *volume stress*. The magnitude of this stress will depend on the **pressure** of the fluid, the force that the fluid exerts on a unit area of a given surface:
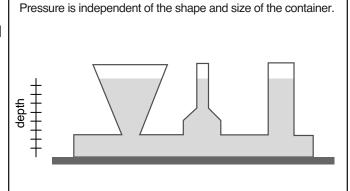
$$P = \frac{F}{A}$$

The SI unit for pressure is the **pascal (Pa)**:

$$1 \text{ Pa} = \frac{1 \text{ N}}{\text{m}^2}$$

Consider a liquid at rest in a container. If we made holes in the bottom and sides of the container, the liquid would pour out of the sides, as well as the bottom. This indicates that the pressure acts sideways as well as downward (it acts upwards as well). Patching our bucket, we could take measurements of the pressure exerted by the liquid at different points and find that the ***pressure increases with the depth*** at which we submerge our instrument. If we were to substitute different liquid substances and then took measurements of the pressure, we would find that ***dense liquids exert a greater pressure than lighter liquids***. One final experiment we might attempt would be to see what effect the shape of our container might have on the pressure of a certain fluid at a given depth. When we begin substituting containers of various shapes, we would discover that at a given depth, ***pressure is independent of the shape and size of the container*** (see illustration below).

If we were patient enough in our analysis of the results of our experiments, we could arrive at the mathematical expression for the pressure, P, at some depth, h, below the surface of a liquid open to the atmosphere. We would find the pressure to be greater than atmospheric pressure by the amount $\rho g h$:
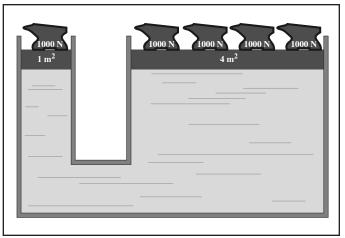
$$P = P_a + \rho g h$$

Pressure is independent of the shape and size of the container.

Pascal's law and the hydraulic press

If we increase the pressure on one of the surfaces enclosing a fluid, this increase will be transmitted undiminished to all parts of the fluid. This law was discovered in the 17th century by Blaise Pascal, a French scientist.

The hydraulic press is one of the most important applications of Pascal's law. With this device, one may multiply forces to almost any desired degree. The apparatus consists essentially of a large and a small piston, each within cylinders connected at the base. The whole vessel is filled with fluid. When a pressure is applied to the small piston, this pressure is transmitted through the fluid and acts on the large piston. If the pistons are at the same height, the pressure on each will be equal, so the force upon each will be directly proportional to their respective areas:
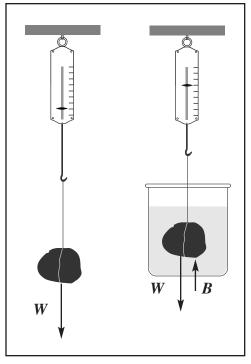
$$\frac{F_a}{A_a} = \frac{F_b}{A_b}$$

The number of times a hydraulic press multiplies the force applied to it is called its *mechanical advantage* or *force ratio*. The hydraulic press in our illustration above has a mechanical advantage of 4. (Note that the terms mechanical advantage and force ratio apply to machines in general, so we could speak of the mechanical advantage of pulley systems, levers, etc..) By conservation of energy, a mechanical advantage which multiplies force is paid for in distance. When the small piston above moves downward 1 meter, the large piston will be raised only 1/4 meter.
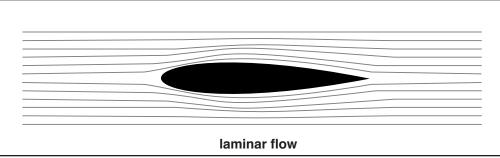
Archimedes' principle: The buoyant force

Archimedes' principle states that a body, wholly or partly immersed in a fluid, will experience an upward force equal to the weight of the fluid displaced by the body. In the illustration at right, we show an object before and after being immersed in water. After immersion, the spring scale registers a downward force that is less by the magnitude of the buoyant force. In our particular example, the buoyant force, **B**, on the object is about half the magnitude of the weight of the object, **W**. Can you estimate the specific gravity of the substance we have immersed?

When an object floats in a given fluid, the buoyant force and the weight of the object are equal. To float, the average density of an object must be less than that of the fluid, because, with only a portion of its volume below the surface, the object displaces a small volume of fluid that weighs as much as the entire object.
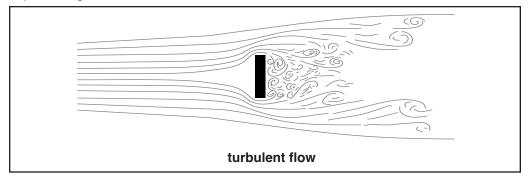
Fluid in motion
The flow of moving fluid can be called either **laminar (streamline)** or **turbulent**.

In laminar flow, the fluid moves in steady parallel layers. Every particle follows the path of the particle before it within a streamline:

laminar flow

Irregular, abrupt changes in the pattern of fluid motion, called eddy currents, characterize turbulent flow, which occurs due to high fluid velocity or under certain conditions which cause abrupt changes in fluid motion.

turbulent flow

Viscosity
Viscosity is an intrinsic property of a fluid indicating the degree of internal friction between the *laminae* or parallel layers of that particular fluid in a streamline. The coefficient of viscosity, η, for a certain fluid is found by computing the ratio of the shearing stress to the rate of change of the shear strain:

$$\eta = \frac{F/A}{v/l}$$

The denominator in the expression above (v/l) is called the velocity gradient. Under a shearing stress, the top layers will move at greater speed than those at the bottom. If the gradient is large for a given stress, then internal friction between laminae must be slight and the fluid is non-viscous. If the gradient is small, then the stress produces a small rate of change of shear strain (the velocity gradient), and the fluid has a large amount of friction between laminae and is viscous.

(Note that the above equation is of the type that does not necessarily call for memorization in preparation for a (unlikely) quantitative problem on the MCAT, but unpacking it and thereby gaining a good sense of the nature of viscosity will benefit your understanding of fluid motion.)

The Reynolds number
A more viscous fluid resists turbulent flow. A quantitative means of predicting whether the flow of a particular fluid will be laminar or turbulent is afforded by the Reynolds number, *RN*:
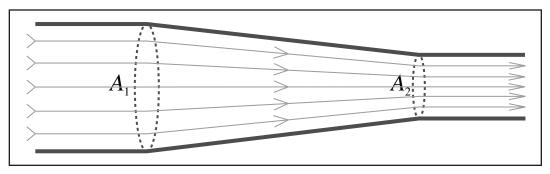
$$RN = \frac{\rho v d}{\eta}$$

ρ is fluid density; v is fluid speed; d is a geometrical property of the flow, such as the diameter of the pipe or the diameter of the obstruction, and η is the viscosity. Turbulence occurs if the Reynolds number is above 3000; laminar flow if it is below 2000.

The flow of an ideal fluid
Certain aspects of fluid motion can be understood if we imagine the existence of an **ideal fluid**: nonviscous, incompressible, and always moving in a steady, streamline fashion (without turbulence).

The equation of continuity
For such an ideal fluid, it must be true that the flow rate through the pipe is constant. Over a duration, an equal volume of fluid will passes through any given cross-sectional area along the pipe. Where the pipe is wide, flow rate is slow. Where it is narrow, the flow rate picks up.

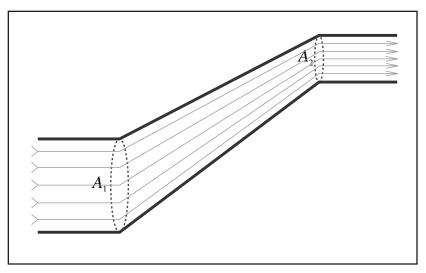The equation of continuity has the quantitative expression:

$$A_1 v_1 = A_2 v_2 = \text{constant}$$

Bernoulli's equation
In the eighteenth century, the Swiss scientist, Daniel Bernoulli, derived the relationship between pressure, fluid speed and elevation.

$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

Bernouli's equation reflects the principle of the conservation of energy for fluid dynamics. The first term, pressure, reflects the amount of work the fluid can perform (If over a length of flow, pressure decreases, the pressure gradient will perform work and accelerate the flow.) The second and third terms correspond to the kinetic and potential energies of the fluid per unit volume. The sum of the three terms is constant for a given position in the streamline.

The relationship expressed by Bernoulli's equation has many interesting applications and consequences. In aerodynamics, airplane wings are designed so that the speed of air across the top surface of the wing is greater than below the wing. Therefore, pressure must be less above the wing than below. This pressure gradient supplies the force to keep the airplane aloft.