Waves

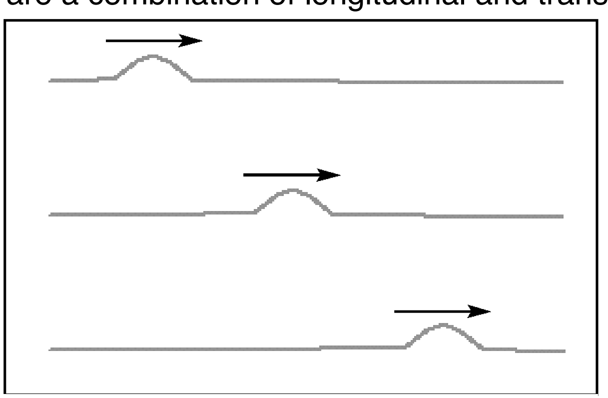
A **wave** is a disturbance traveling through a medium. When we speak of wave motion, we are not speaking of matter in motion as we do in kinematics; we are speaking of the motion of energy.

To get a sense of how, as a wave, energy propagates through a medium, we shall begin our discussion with wave motion on the surface of a body of water. Eve ryone is familiar with the circular, widening ripples that appear upon a pool of water into which a pebble has been dropped. The pebble collides with the surface of the water, pushing the water downward. The water is not compressed, however, so immediately around where the pebble has fallen, a circular wall of water rises up to make room for the water that was pushed downward. When our pebble was falling, it possessed an amount of kinetic energy, some of which has now been transfo rmed into the potential energy of lifted fluid. But it doesn't stop there. The lifted fluid itself then falls, and when it does, it falls below the level of the original surface of the pool. This lifts water again to make room both inside and outside of of the perimeter of what just fell. The wave is propagating. The water lifted outside falls, causing more water to be lifted beyond it. The water lifted inside, where the pebble originally fell, falls too, causing another ripple to begin. Water in the spot where the pebble originally fell thus oscillates up and down, sending ripples outward, one after the other, decreasing in energy. From above, it looks as though the water itself is moving outward, but it is energy moving outward. The water molecules in fact move in small circles, as successive waves pass across the surface of the pool.

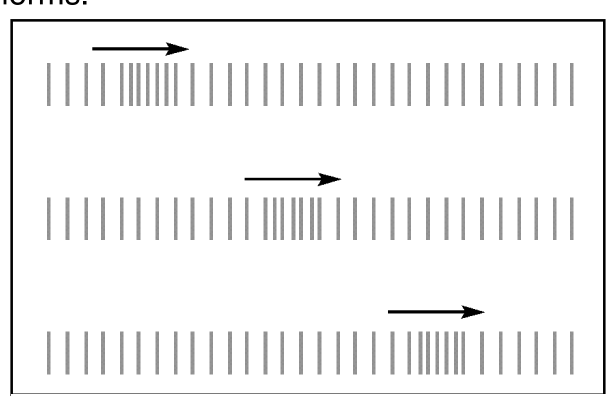
The type of wave we are discussing above is a **harmonic wave**, which means that the wave form repeats itself with successive ripples. This is due to the oscillatory nature of the portion of water into which the pebble was dropped. A wave also can consist of a single *pulse*, which does not repeat itself. This type of wave called a **traveling wave.**

Traveling waves: transverse and longitudinal

Waves move across the surface of a pool of water as successive disturbances, and the molecules of water oscillate in a circular manner, at times perpendicular and at times parallel to the direction of wave motion. Waves in which the particles of the medium move in a direction purely *perpendicular* to wave velocity are called **transverse** waves. A *transverse traveling wave* is pictured below left. A single pulse, produced on a length of rope with one quick shake, moves through the medium. As the disturbance approaches a given segment of rope, that segment rises up, and as it leaves, the segment falls back down. At bottom right, however, is pictured a **longitudinal** traveling wave, such as might be produced in a stretched "slinky". Here the motion of the medium is *parallel* to wave velocity. As the disturbance approaches a given segment of the spring, the segment begins to compress, and as it leaves, the segment relaxes back to equilibrium. Waves across the surface of water are a combination of longitudinal and transverse forms.



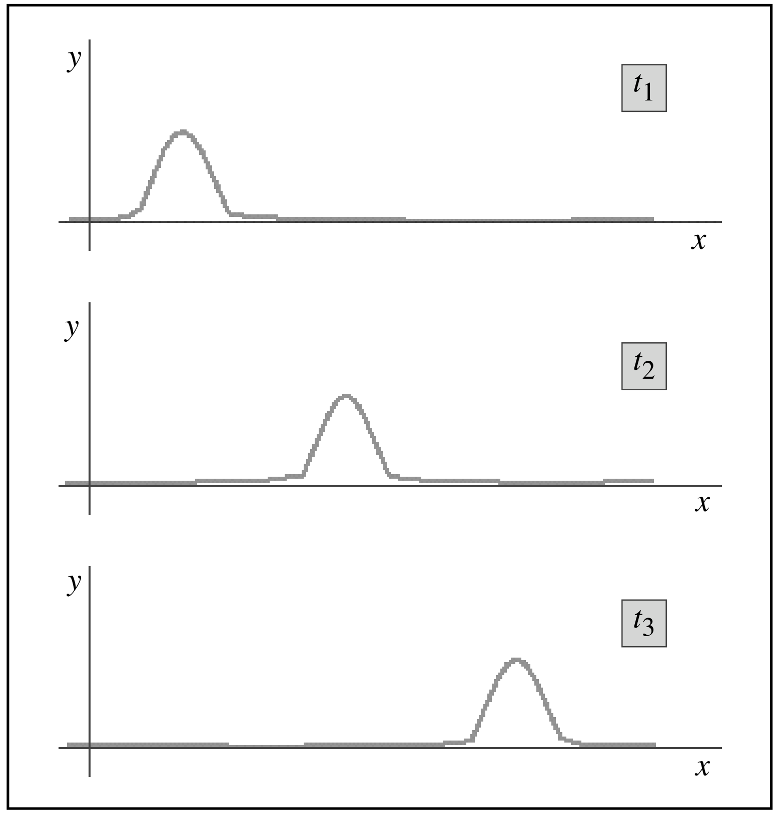
transverse traveling wave



longitudinal traveling wave

Describing waves with math: wave functions
On the MCAT, it is unlikely that you will have to perform any complicated mathematical operations in which you are asked to generate the wave function describing this or that wave. It's a good idea, however, to understand how wave functions work to describe waves in case the test-makers give you one to interpret. Also, as is often the case, an understanding of how math can be used to describe a certain phenomenon will increase your degree of understanding.

At right we have represented a transverse pulse traveling along our length of rope within a cartesian coordinate system. The x axis corresponds to position along the rope, and the y axis corresponds to the rope's vertical displacement. We have three graphs, corresponding to three successive moments in time, t_1, t_2, and t_3. The vertical displacement of a point along the rope can be thought of as a function of both position in the medium and time. The general expression for a wave traveling to the right is:

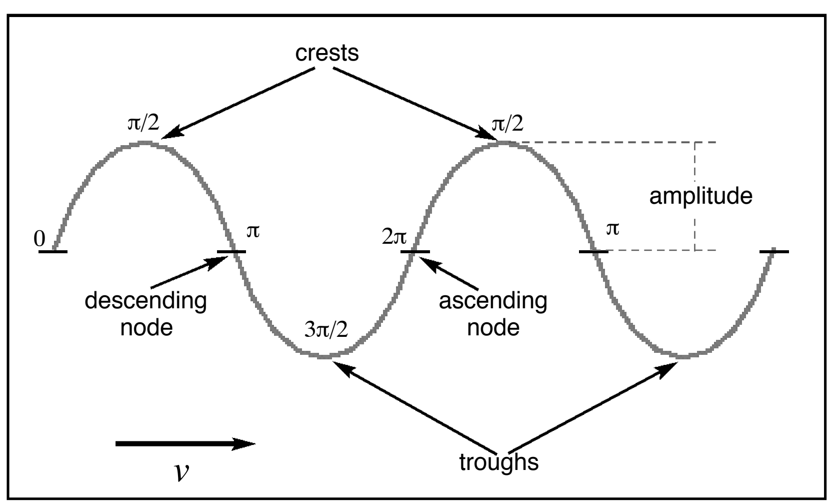


$$y = f(x - vt)$$

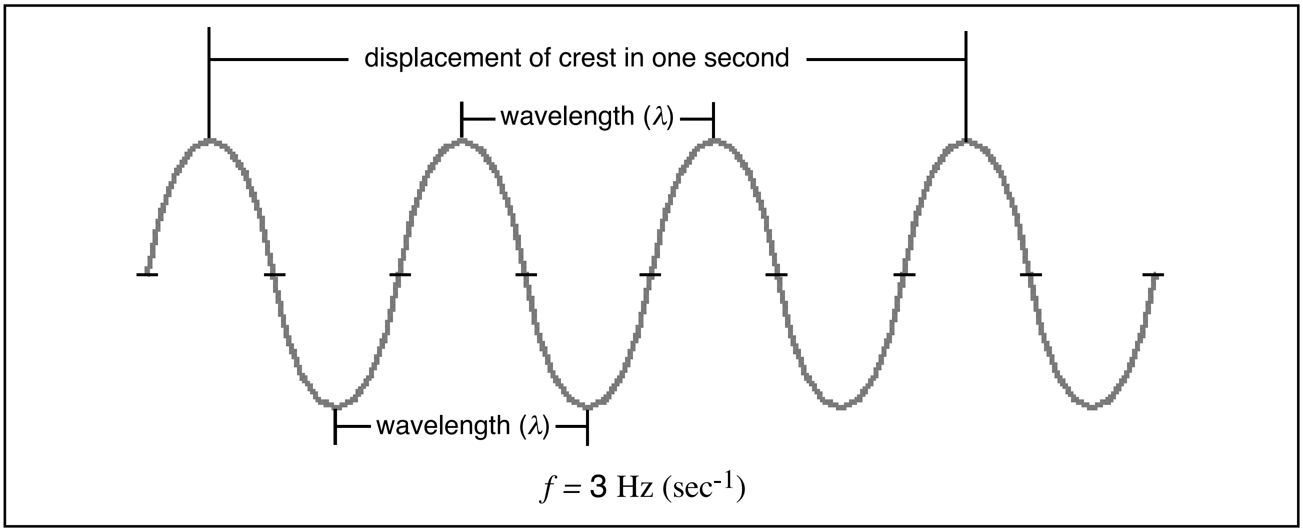
where v is the **wave speed**. A wave travelling to the left would have the function:

$$y = f(x + vt)$$

Harmonic waves
A harmonic wave is one in which the wave form exhibits a periodic nature. Such waves are represented by the sine curve. Thinking back to our ripples on a pool of water, examine the sine wave below. Imagine the wave fo rm moving to the right and think about the particles of water moving up and down and then up again as the wave passes (It must be admitted that the actual movements of the water particles are more complicated, the particle motion being a combination of transverse and longitudinal forms).
The points where the disturbance is greatest in the upward direction are called *crests*, where the disturbance is greatest dow n-ward, we have *troughs*.
Where the water is at norm al level are *nodes*, which can be either *ascending* or *descending*, depending on whether the water is rising or falling at that point.
The vertical distance from a node to a crest or to a trough is called the *amplitude* of the wave.



The wavelength and frequency of harmonic waves



Harmonic waves are periodic, meaning the wave form repeats itself. The distance within a harmonic wave between successive points where the nature of the disturbance is identical (crest to crest or trough to trough, for example) is called the **wavelength (λ)**, measured in meters. The number of crests which pass a certain point in a second is called the **frequency (f)**, measured in Hz or sec^{-1}. The **wave speed (v)** is the product of the wavelength and frequency:

$$v = \lambda f$$

The frequency (f) tells you how many crests pass by a given point in one second. The reciprocal of this quantity tells you how many seconds it takes for one wavelength to pass by (how many seconds per cycle), which is called the **period (T)**, measured in seconds.

$$T = \frac{1}{f}$$

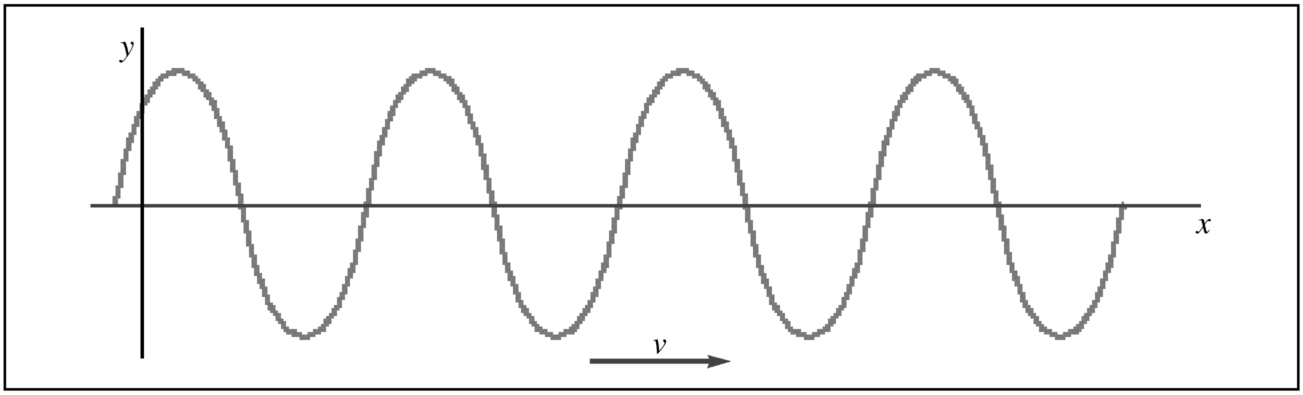
You can describe a harmonic wave in terms which more directly reflect the sinusoidal nature of the wave form. Each wavelength corresponds to progress from 0 to 2π radians to complete a cycle (At 2π radians, the wave begins a new cycle). The quantity called the **angular frequency (ω)**, which is measured in radians per second (rad sec^{-1}), tells how quickly a wave oscillates in terms of angular progress. The harmonic wave in the illustration at the top of the page has a frequency of 3 Hz, or 3 cycles per second. In one second, 6π radians of angular progress will accrue. The angular frequency is thus 6π radians per second (3 cycles per second multiplied by 2π radians per cycle).

$$\omega = \frac{2\pi}{T} = 2\pi\ f$$

Another useful quantity is called the **wave number (k)**, which is measured in radians per meter (rad m^{1}). Imagine holding the harmonic wave still in time. If you break the wave up into meter segments, the amount of angular progress in one meter is the wave number.

$$k = \frac{2\pi}{\lambda}$$

The harmonic wave function



The wave function for a harmonic wave is given by:

$$y = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right] = A \sin(\mathrm{kx} - \omega \mathrm{t})$$

Unpacking the formula might help us understand how such a function works to represent a harmonic wave. First of all, the quantity A is called the *amplitude* of the wave. When the sine equals 1 or -1, the wave will have a magnitude of displacement equal to this quantity A. At such moments in time and space, the wave will be passing through a crest or a trough.

Using our imagination, let's investigate the expression within the sine function. We find two terms. One term is a function of horizontal displacement, the other a function of time. Holding time constant in your imagination, vary the displacement, x, in increments of one wavelength. As you do so, examine the equation and notice that the addition of one wave-length will move the wave 2π radians into the next cycle. Therefore, the coefficient in the displacement term, which depends on the wavelength, ($2\pi/\lambda$ or k) specifies the shape of the wave in space, whether the wave looks compressed or stretched out. A large wave-length means that there will be a large distance along the x-axis for each cycle.

Next, in your imagination, examine the equation and hold the displacement constant at a single point and move time forward in increments of the period, T (remembering that the frequency, f, is equal to v/λ). Each of these additions of one period takes 2π radians away from the sum within the sine function. Do you see how increasing the time by one period brings the disturbance one cycle behind our point along the x-axis up to where we are? A high frequency, f, means that the wave will move many cycles forward in a small amount of time.

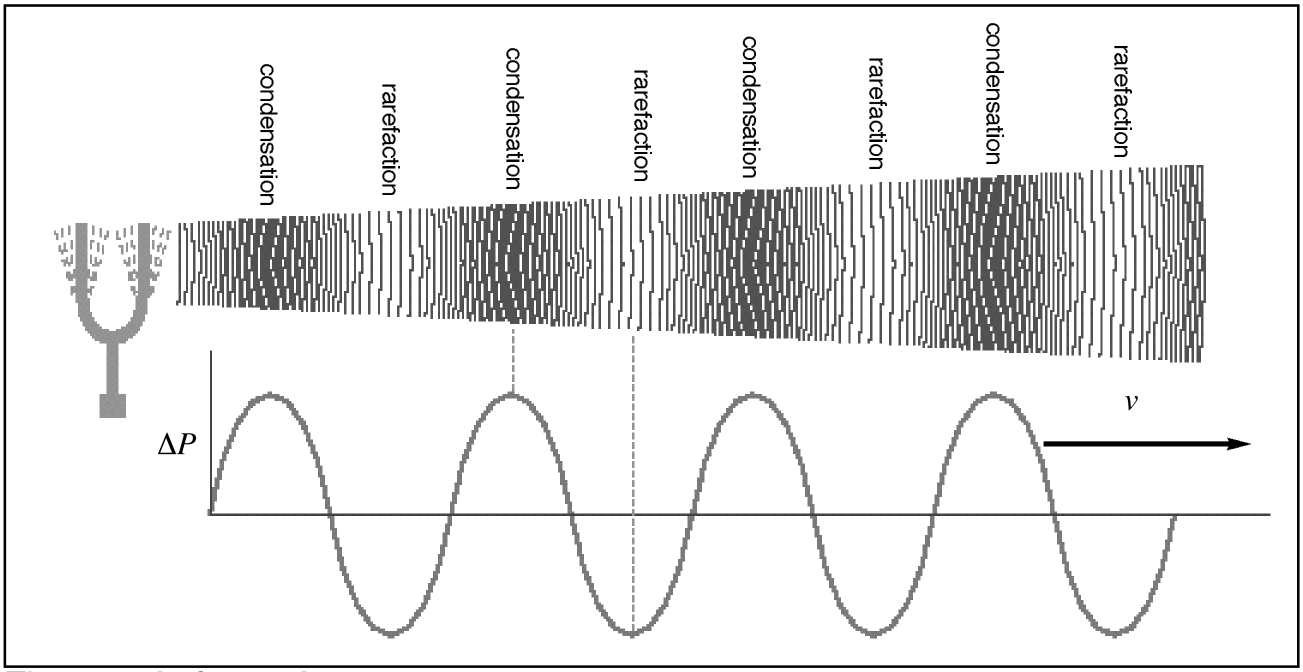
This type of mental exercise upon a physical formula, holding certain terms constant and varying others to see how some phenomenon varies with the quantitative parameters that go into describing it, develops the kind of sophisticated understanding and appreciation of mathematical physics that can make crucial difference on the MCAT. Really it should get to be a habit. Our conclusion in this instance, with the harmonic wave function, should be that while the displacement term describes the shape of the wave, the time term puts the wave in motion.

One final note: Sometimes the wave function will show the addition of a constant within the sine function. This term, ϕ, is called the **phase constant**, which is specified by the initial conditions (we might not start timing at the beginning of a cycle, for example)

$$y = A \sin(\mathrm{kx} - \omega \mathrm{t} - \phi)$$

Sound waves

The tines of a tuning fork will vibrate with harmonic motion. The interaction of the tines with the fluid medium in which they are immersed (in this case, the air) produces a series of high and low pressure regions called **condensations** and **rarefactions**. The mathematical description of a harmonic sound wave would be identical to that which we have described earlier for harmonic waves in general, except that in this case we are describing the displacement from equilibrium of air molecules in the horizontal direction. Sound waves are not transverse but **longitudinal**. As molecules of air are displaced, there will appear sections in which the air is compressed and sections in which the air is expanded. 90^o out of phase with the displacement wave is the pressure wave, which we have shown below:



The speed of sound waves

As a general rule, *the speed of waves through a medium depends only on the properties of the medium*. For example, the speed of a transverse wave along a stretched string is given by:

$$v = \sqrt{\frac{F}{\mu}}$$ speed of wave along a stretched string

In the formula above, F is the tension on the string and μ is the mass per unit length. A disturbance occurs upon a section of string and the disturbance begins to propagate. The time it takes for a particular, displaced segment to transmit the disturbance to the following segment will be short if the restoring force, which depends upon the tension, F, is large. Also, if the inertia of the segment is large, which depends on μ, the time it takes to transmit the disturbance will be long, because it will take more force to return the segment to equilibrium.

The same principles apply to the speed of sound, which is given by the formula:

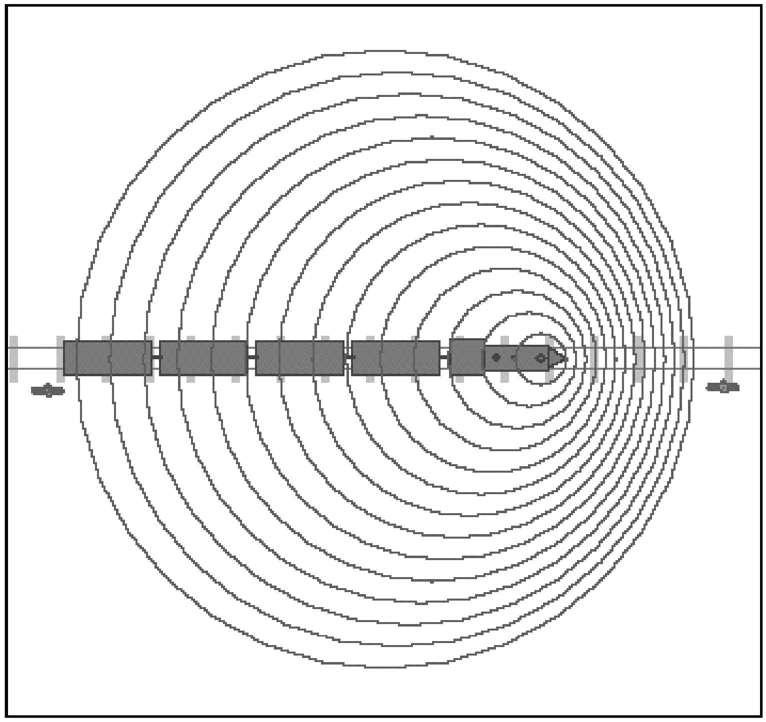
$$v = \sqrt{\frac{B}{\rho}}$$ speed of sound in a particular medium

B is the bulk modulus of the medium, which will determine the restoring forces which accompanies a deformation (condensation or rarefaction), and ρ is the equilibrium density of the medium.

The speed of sound through air at O ^{o}C is **331 m/s**.

The Doppler effect

There is a change in the apparent frequency of sound whenever there is relative motion between observer and source. This is called the **Doppler effect**. If the source and observer are moving towards each other, the observer hears the frequency as higher than the frequency of the source. If the source and observer are moving away from each other, the sound appears to be of lower frequency. The general equation for determining the apparent frequency, f, is as follows:

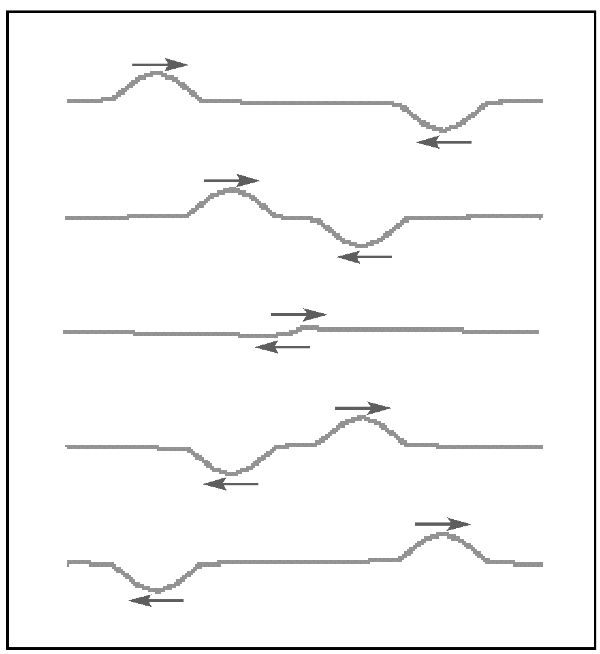


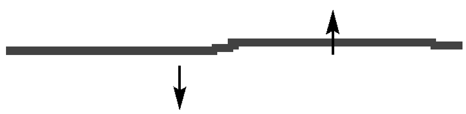
$$f' = f\left(\frac{v \pm v_0}{v \mp v_s}\right)$$

Notice that there is a choice of signs, governing whether to add or to subtract the speed of the observer, v_0, to the speed of sound, v, in the numerator, and also whether to subtract or add the speed of the source, v_s, to the speed of sound in the denominator. Don't let this confuse you. Use the upper signs, *which will increase the apparent frequency*, if the observer and source are moving toward each other, and use the lower signs if they are moving apart.

Wave superposition

Interference describes the occurrence of waves occupying the same region of space and combining to produce a resultant wave. The amplitude of the resultant wave is the sum of the amplitudes of the individual waves. At positions where the displacements of the individual waves are in the same direction (both positive or both negative), **constructive interference** occurs, and the two waves reinforce one another to produce a resultant wave of greater amplitude than either alone. At positions where the displacements of the individual waves are in opposite directions, the waves act to cancel each other out. When this occurs it is called **destructive interference**. At right, we have illustrated the destructive interference of two transverse traveling waves of equal amplitude but of opposite orientation. Notice at the position of overlap, it appears that the pulses disappear momentarily. What happens to the energy during overlap? We know that it is conserved. What form is it in? The answer is that all of the energy of the wave is in the form of kinetic energy. The direction of the transverse motion of the string on either side of the resultant of the two pulses is in opposite directions:





Standing waves in a fixed string

A harmonic wave moving on a stretched string fixed at both ends will be reflected upon encountering either end. The net result upon the string will be the superposition of two harmonic waves moving in opposite directions. This superposition creates a resultant wave, called a **standing wave**, in which the amplitude is a function of horizontal position. The resultant wave function is as follows (for the MCAT it is more important to understand this than memorize):

$$y = (2A_0 \sin kx)\cos \omega t$$

The wave function predicts **nodes**, points with zero amplitude, when the following is true:

$$kx = \pi, 2\pi, 3\pi, \ldots$$

Since $k = 2\pi/\lambda$, the nodes in terms of the wavelength can be expressed (n = 1,2,3...):

$$x = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots \frac{n\lambda}{2}$$

Because the nodes are points along the string that have zero amplitude, it is natural for nodes to be located at the fixed ends. These natural ways for the string to vibrate are called its **normal modes**. The previous equation then becomes:

$$L = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots \frac{n\lambda}{2}$$

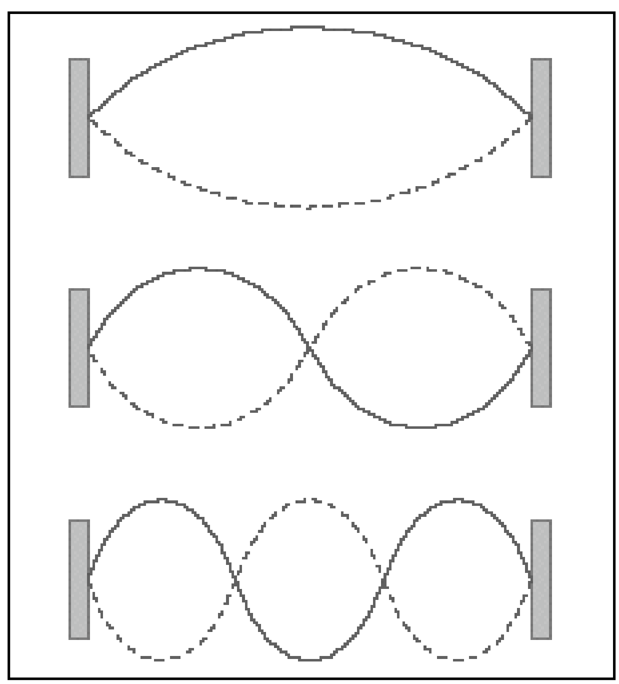
The first three normal modes are illustrated in the picture below right. The wavelengths of the normal modes may be expressed:

$$\lambda_n = 2L, L, \frac{2L}{3}, \ldots \frac{2L}{n}$$

With the wave speed, v, we generate the expression for the frequencies of the normal modes:

$$f_n = \frac{v}{\lambda_n} = \frac{n}{2L}v$$

The frequency for n = 1 is called the *fundamental frequency*, which together with the other terms in the series constitute a *harmonic series*. When an initial disturbance is introduced, such as the occurrence of the collision of a hammer upon a string of a piano, it doesn't matter where upon the string the blow is struck. Waves of other than the normal modes of vibration will cancel themselves out by destructive interference, and the string will vibrate with the frequencies described by its harmonic series. It will vibrate will all of the frequencies in the harmonic series at once, a resultant wave being created by superposition of the normal mode wave forms. We call the second harmonic the first *overtone*. The third harmonic is the second overtone, etc..

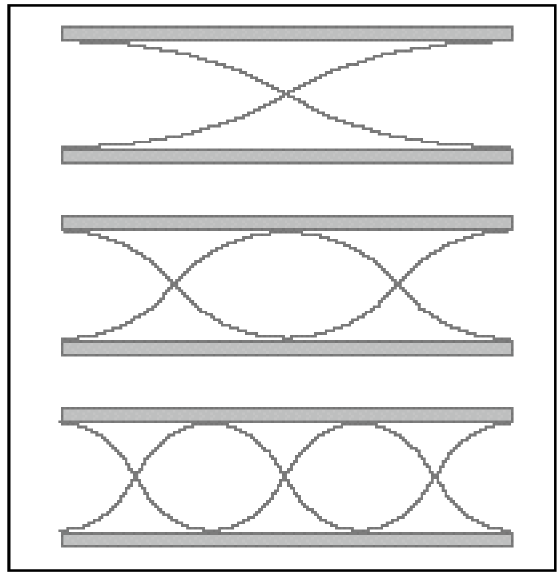


Standing waves in air columns

Columns of air, such as those found in in a pipe organ, can be used to create standing sound waves. The description of the standing wave's harmonic series depends upon whether the pipe is open at both ends or closed at one end.

A pipe open at both ends:

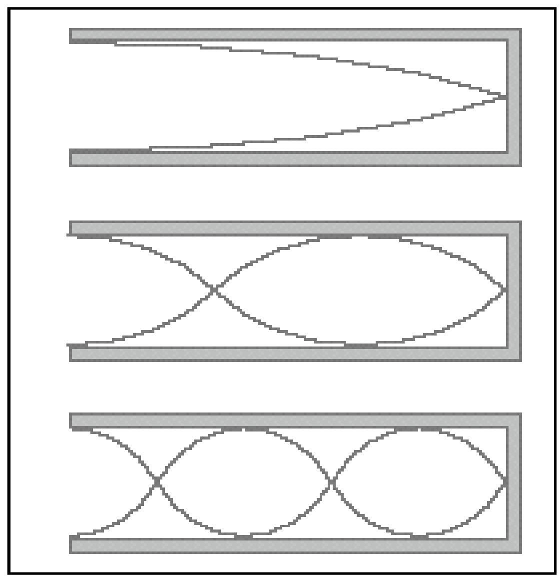
A sound wave reflecting at the open end of the air column is encountering outside air. The outside air has "give" to it (the acceptance and return of a pulse takes a full cycle), so the reflected wave will be in phase with the incident wave. This means that the open end of an air column is a displacement crest or trough (a pressure node) in the standing wave. The natural frequencies of vibration have the same description as those of a stretched string of the same length (except, of course, that the wave speeds, v, are differently determined):



$$f_n = \frac{n}{2L}v \quad (n = 1,2,3,\ldots)$$

A pipe closed at one end:

When a sound wave is incident upon a solid surface with little "give" to it, it reflects 180° out of phase. The reaction force exerted by the wall returns the wave upon itself with equal and opposite displacement. The net result by superposition is that a displacement node exists at the closed end. From the previous discussion, we know that a displacement crest or trough (antinode) will exist at the open end. Therefore, the fundamental frequency of a pipe closed at one end will reflect a wavelength four times longer than the pipe:



$$f_n = \frac{n}{4L}v \quad (n = 1,3,5,\ldots)$$

Resonance

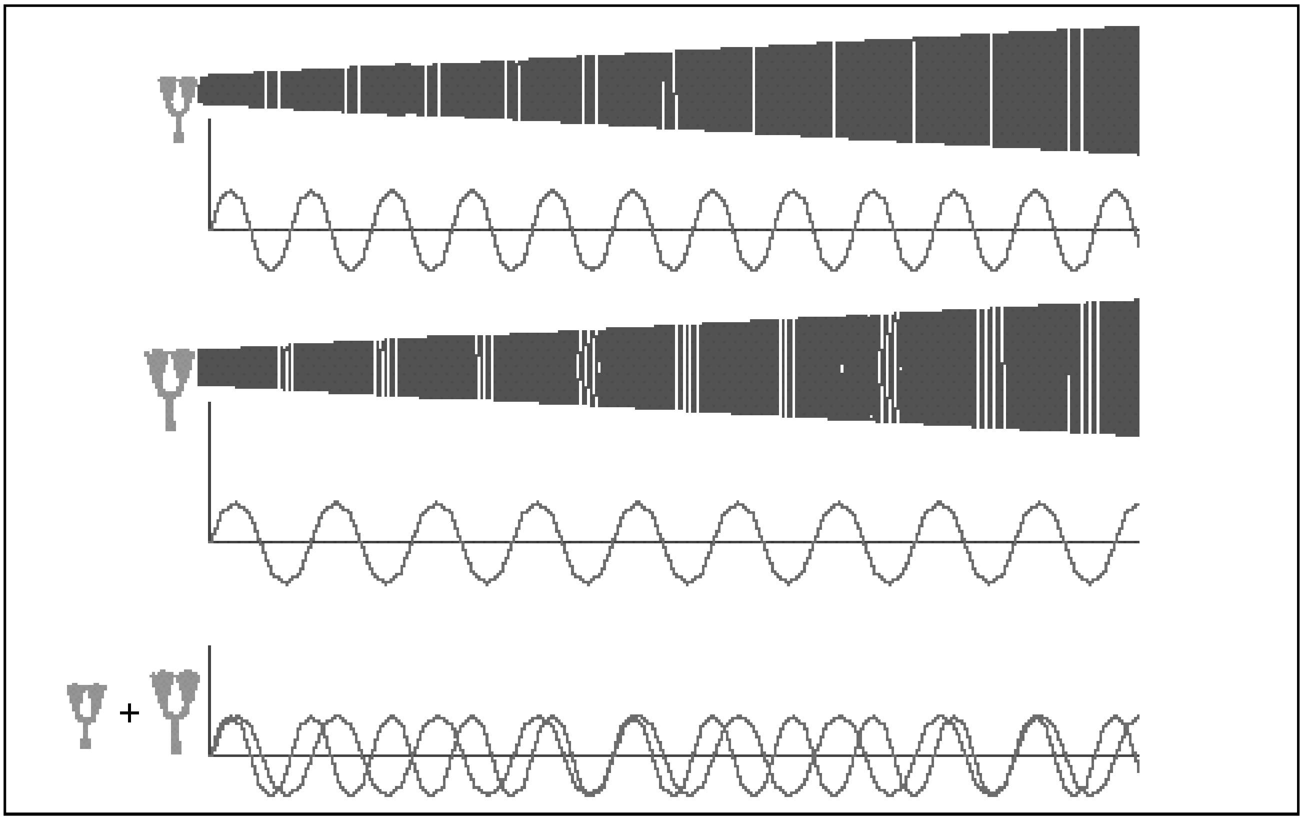
Resonance between an oscillatory system and a driving force exists when the frequency of the driving force corresponds to the frequency of the system. A child pushing another on a swing will work naturally to achieve resonance to swing their friend as high as possible.

One type of resonance known as *sympathetic vibration* can occur between two identical tuning forks set side by side. When one is struck, it sends out sound waves that correspond to the natural frequency of vibration of the other. The varying pressure of the sound waves exerts a resonant force upon the other tuning fork, and after a time, it too will begin to generate sound waves.

The term resonance is also used to describe a situation where direct sound waves are reinforced by reflected sound waves, such as occurs when the sound from the strings of a guitar are reinforced by the (complex) air column in the body of the instrument.

Beats

When two tuning forks of slightly different frequencies are played together, at a given position in space, the sound waves are periodically in phase, resulting in constructive interference, and then out of phase, resulting in destructive interference. A regular series of sounds of pulsating intensity are heard, called **beats**. Examine the illustration below to see how this occurs.



The number of beats one hears per second, the **beat frequency**, is simply the difference between the frequencies of the vibrating bodies:

$$f_b = f_1 - f_2$$

Suppose in the illustration above the waves achieved in one second the total horizontal displacement represented by the length of the graph. In that case their frequencies would respectively be about 12 and 9 cycles per second. Can you see the three beats in the graph representing their superposition?

The intensity of sound, the decibel scale

The **intensity** of a sound wave is a measure of the amount of energy in unit time moving through a unit area, or, in other words, the power flowing through a unit area. The **loudness** of sound is a subjective measure that reflects its intensity. However, the way we perceive gradations of loudness is closer to the *logarithm* of the intensity. In other words, we would perceive a sound ten or a hundred or a thousand times as intense as only an arithmetic (1,2,3), not a geometric, increase. A scale of measurement was developed, called the **decibel scale**, to bring measurements of intensity into line with our perceptions of loudness. The *intensity level*, β, measured in decibels can be found with the following formula:

$$\beta = 10 \log \left(\frac{I}{I_0}\right)$$

I is the intensity of our sound wave, measured in W/m^2. I_0 is the *reference intensity* (10^{-12} W/m^2), the threshold of hearing.