Physics Practice

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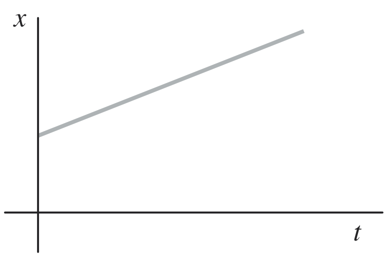
Kinematics Practice Items

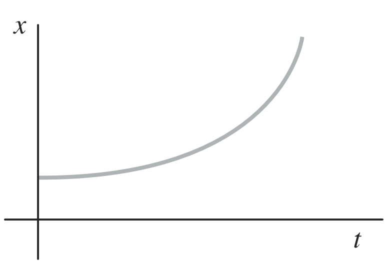
1. Which of the following is not a vector quantity?

A. The velocity of a golf ball in flight
B. The distance from Atlanta to Dallas
C. The acceleration of a feather in free fall through a vacuum
D. The displacement of a fluid particle from one end of a pipe section to the other

2. Which of the displacement-time graphs below represents constant positive velocity?

A.


B.


C.


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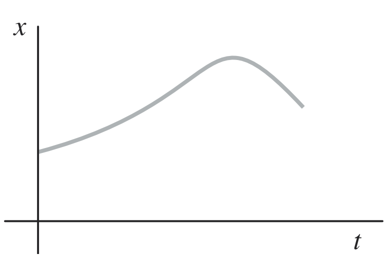


3. Speed...

A. is the magnitude of the change per unit time of the velocity vector.
B. can never be negative.
C. is equivalent to the slope of the line tangent to the displacement curve.
D. has both magnitude and direction.

4. How long does it take a ball thrown straight upwards at 10 m/s to reach its maximum height (neglecting air resistance)?

A. 5 s
B. 0.5 s
C. 1 s
D. 10 s

5. An automobile travels in a straight line for 10 seconds at 20 m/s then accelerates uniformly to a speed of 30 m/s in the next 10 seconds. Find the total displacement.

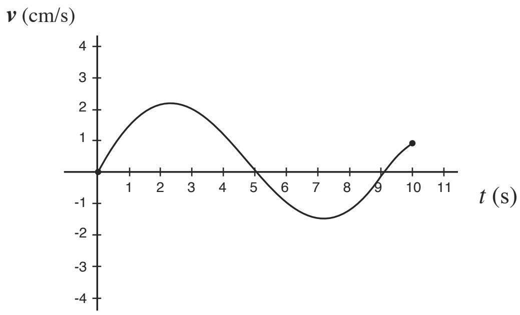
A. 450 m
B. 500 m
C. 550 m
D. 800 m

6. At time t = 10s a dragster is moving in a straight line with a velocity of 80 m/s. At t = 20s its velocity is 120 m/s. What is the average velocity of the dragster for the interval 10s to 20s?

A. 95 m/s
B. 100 m/s
C. 110 m/s
D. cannot be determined from given information

The graph below pertains to questions 7-9.

The graph was derived from measurements of blood velocity within the port of a hemodialysis catheter carried out over ten seconds.



7. At which approximate moment in time did the blood cease moving forward and begin moving backward in the catheter?

A. 2.2 s
B. 5.0 s
C. 7.2 s
D. 9.1 s

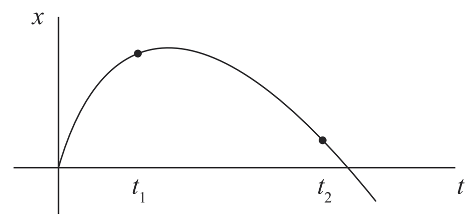
8. At which approximate moment in time did a volume element in the blood experience an acceleration equal to zero?

A. 0 s
B. 5.0 s
C. 7.2 s
D. 10 s

9. Which of the following is the nearest approximation of the average acceleration of a volume element within the blood during that time period?

A. 0.001 m/s^2
B. 0.1 m/s^2
C. 1 m/s^2
D. 10 m/s^2

10. For the one dimensional motion of a particle, the curve below shows displacement vs. time. Which of the following is the best description of the motion within the time interval t_1 to t_2?



A. The particle attains maximum speed then returns to its original position.
B. The particle attains maximum speed then gradually decelerates.
C. The particle comes to rest then moves away from its original position.
D. The particle comes to rest then moves with negative velocity towards its original position.

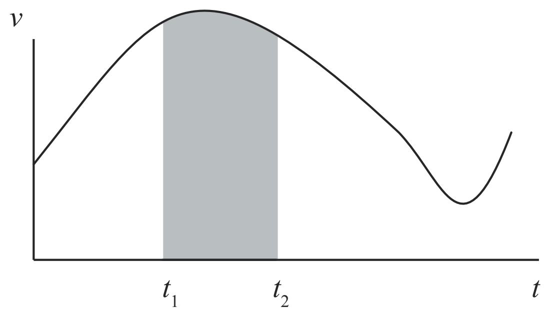
11. A car uniformly increases its speed from 30 m/s to 50 m/s over a distance of 400 meters on a straightaway. What is the magnitude of acceleration?

A. 0.5 m/s^2
B. 2 m/s^2
C. 5 m/s^2
D. 4 m/s^2

12. What is the vertical component of the velocity of a sky-diver 10 seconds after jumping? Disregard air resistance.

A. -25 m/s
B. -50 m/s
C. -100 m/s
D. -500 m/s

13. For a particle in one dimensional motion, the shaded area beneath the velocity vs. time curve below corresponds to:



A. the average velocity of the particle during the time interval t_1 to t_2
B. the displacement the particle over the time interval t_1 to t_2
C. the average speed of the particle during the time interval t_1 to t_2
D. the acceleration of the particle during the time interval t_1 to t_2

14. Which of the following statements is true concerning an object undergoing uniform circular motion?

A. The velocity and acceleration vectors are always perpendicular.
B. The object moves with constant speed.
C. The direction of the acceleration vector is towards the center.
D. All of the above are true statements.

15. Over a distance of 3 cm an electron accelerates uniformly from a speed of 9×10^3 m/s to a speed of 6×10^6 m/s. How long does it take the electron to cross this distance?

A. 1.0×10^{-8} s
B. 3.6×10^{-8} s
C. 5.0×10^{-7} s
D. 1.0×10^{-6} s

16. The velocity of a particle moving along a line is described by this expression:

$$v(t) = \frac{1}{3}\left[(t-1)^{(t+1)}\right]$$

Find the average acceleration of the particle during the time interval t = 1 s to t = 4 s.

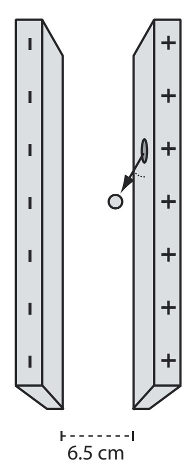
A. 0.3 m/s^2
B. 27 m/s^2
C. 81 m/s^2
D. Cannot be determined from given information.

17. A projectile is launched from ground level over a horizontal plane on the surface of the Earth at sea level at a 30^o angle to the horizontal. Neglecting air resistance, what other information is needed to calculate the range of its trajectory?

A. initial speed
B. initial speed, mass
C. the vertical component of the velocity
D. the initial kinetic energy of the projectile

The following pertains to questions 16 - 18.

If the size of oppositely charged parallel conducting plates is large compared to the distance between the plates, then the plates can be treated like infinite planes (neglecting fringing), and the electric field is the same everywhere between the plates. No matter where a charge is placed between to oppositely charged parallel plates, the net force on that charge is the same, and it will experience uniform acceleration.



An electron enters the region between two parallel electrically charged metal plates with an initial velocity of 6.0×10^5 m/s at an angle of 30º to the positive plate as shown in the figure at right. The electron experiences a constant acceleration of 1.5×10^{12} m/s^2 to the right.

18. The electron follows a path that is

A. parabolic
B. circular
C. a straight line
D. composed of two straight line segments

19. How long will the electron be in flight?

A. 2.0×10^{-8} s
B. 3.4×10^{-8} s
C. 2.0×10^{-7} s
D. 4.0×10^{-7} s

20. Assume the electron will be in flight for 4.0×10^{-7} s. What is the minimum distance at which the two plates may be spaced to prevent the electron from striking the negatively charged plate?

A. 1.2 cm
B. 2.4 cm
C. 3.0 cm
D. 6.0 cm

Kinematics
Answers and Explanations

1. B

A quantity is either a vector or a scalar. Vectors are quantities that are fully described by both a magnitude and a direction. Displacement, velocity, and acceleration are vector quantities. In contrast, distance is a scalar quantity. Scalars are quantities that are fully described by a magnitude alone. The distance from Atlanta to Dallas is an equal distance as that from Atlanta to Philadelphia. However, to go from Atlanta to Dallas is a different displacement than to go to Philadelphia. Those are different changes in position. Additionally, it's good to note also that while velocity is a vector quantity, speed is a scalar quantity. Speed is the magnitude of the velocity. If we were driving toward Dallas at 70 miles per hour, that would be the same speed if we were driving 70 mph towards Philadelphia. However, because Dallas is towards the West from Atlanta and Philadelphia is towards the Northeast, those would be different velocities.

2. B

Velocity is the rate of change of the displacement. The slope of the displacement vs. time curve. The change in x divided by the change in t. The graph of displacement with a constant positive velocity is a curve with a constant positive slope, or, in other words, a straight line sloping upwards.

3. B

In the kinematics of one dimensional motion, ie. motion on a line, we use the arithmetic positive and negative to signify direction. Positive means upward vs. negative means downward, or positive means rightward vs. negative means leftward. You can do this because adding and subtracting vectors on a line is just simple arithmetic. However, this sense of a kinematic quantity possibly being negative only applies to vector quantities. Velocity has direction. It does not apply to speed. Speed is the magnitude of the velocity. Whether a body is moving to the right to the left does not signify with regard to its speed, which being a magnitude, can only be positive.

4. C

The acceleration due to gravity is 9.8 m/s^2 directed downward. Just use 10 m/s^2 for the MCAT. For a question like this, you don't want to have to go to a formula. Just think about what 10 m/s^2 means. The velocity is changing ten meters per second per second. Every second the velocity of the ball will change by ten meters per second. For a ball thrown upwards you ask yourself, how long will it take the acceleration due to gravity to deplete the initial upward velocity to zero? If we started upwards at 10 m/s and gravity is going to change that by 10 m/s every second, it's going to reach the peak in 1s.

5. A

With this problem we should treat each 10 s interval separately. For the first 10 s the bus is traveling at a constant velocity, so the displacement is simply the product of this constant velocity and the time. A rate of change times a duration of time equals an amount of change:

$$x - x_0 = v\,\Delta t$$

$$200\text{ m} = (200\text{ m/s})(10\text{ s})$$

For the second 10 s interval, the bus accelerates uniformly to 30 m/s. The displacement equals the product of the average velocity and the time. Because the acceleration is uniform, the average velocity is simply the arithmetic mean of the initial and final velocities:

$$x - x_0 = \frac{1}{2}(v + v_0)t$$

$$250\text{ m} = \frac{1}{2}(20\text{ m/s} + 30\text{ m/s})(10\text{ s})$$

The sum of the displacements of the two intervals is the total displacement: 450 m.

6. D

There is not enough information here to determine average velocity. If the acceleration were constant we could do it, just take the arithmetic mean of the initial and final velocities. However, the dragster had greater acceleration at the very end.

7. B

Read the axes, legends and captions. Mentally picture what's happening as if it were in front of you. At 5.0 s the blood transitions from a positive velocity to a negative velocity. It had been moving forward and now it is moving backward.

8. C

The acceleration is the slope of the velocity versus time curve. This occurs twice on the curve, at 2.2 s and 7.2 s.

9. A

On the velocity vs. time curve, the blood in the catheter demonstrates a fairly complex pattern motion. It moves forward, speeding up. Then it begins to slow down and reverses course for a period of time, moving backwards. The acceleration is the slope of this curve, and you can see it is constantly changing. Even though the motion is complex, the answer to question of the average acceleration is not hard to determine. The change in the velocity equals the product of the average acceleration and the time interval, or, in other words, average acceleration equals the change in velocity divided by the time:

$$\bar{a} = \frac{v - v_0}{\Delta t}$$

Remember to convert cm/s to m/s! Many MCAT questions are actually tests of focus and attention. Be careful to convert to S.I. units.

$$v - v_0 = 1 \text{ cm/s} = 0.01 \text{ m/s}$$

$$\Delta t = 10 \text{ s}$$

$$\bar{a} = 0.001 \text{ m/s}^2$$

10. D

The velocity is the slope of the displacement vs. time curve. At t_1 the particle is moving with a positive velocity. At the peak of the curve the particle has zero velocity. It comes to rest and begins moving back towards its original position with a negative velocity.

11. B

Crucially, we are told that acceleration is constant. We're given an initial and final velocity and a total displacement. We know that the displacement equals the product of the average velocity and the time, and we know that when acceleration is constant, the average velocity is the simple arithmetic mean of the initial and final velocities. This gives us an easy path to determine the time interval, and if we know the time interval as well as the change in velocity, we can determined the acceleration.

$$x - x_0 = \tfrac{1}{2}(v + v_0)t$$

$$t = \frac{x - x_0}{\frac{1}{2}(v + v_0)} = \frac{400 \text{ m}}{\frac{1}{2}\,(30 \text{m/s} + 50 \text{ m/s})} = 10 \text{ s}$$

$$a = \frac{\Delta v}{\Delta t} = \frac{(50 \text{m/s} - 30 \text{ m/s})}{10 \text{ s}} = 2 \text{ m/s}^2$$

Alternatively, we could have used the following equation to find the answer more directly, but, perhaps, less intuitively:

$$v^2 = v_0^2 + 2a(x - x_0)$$

12. C

The acceleration due to gravity is -10 m/s^2. In plain English, gravitation changes the velocity 10 meters per second each second. After ten seconds, that will mean a change in the velocity of 100 meters per second. For constant acceleration, this idea is just a simple variation of the idea that an amount of change equals the rate of change times the duration of time:

$$\Delta v = a\,\Delta t$$

13. B

The area under the curve of a rate of change is the amount of change. In other words, the area under the velocity vs. time curve is the displacement.

14. D

All of the statements are true. An object undergoing uniform circular motion moves with a constant speed in a circular path. Although the speed is constant, the velocity is constantly changing in its direction. The velocity vector is tangent to the circular path while the acceleration vector points towards the center.

15. A

First thing to do is reconcile our units in the SI system. We were given the displacement as 3 cm, so we should convert this to meters. Also, because we can see arithmetic involving scientific notation in our future, we should convert it to meters in scientific notation. These kind of set up steps at the start are a good way in problem solving to do something useful while your unconscious mind accommodates itself to the problem.

$$3\text{ cm} = 0.03\text{ m} = 3 \times 10^{-2}\text{ m}$$

We were given an initial and final velocity and told that acceleration is constant. In that case, we know the average velocity will be simply the arithmetic mean of the initial and final. The product of the average velocity and the time interval equals the displacement.

$$x - x_0 = \frac{1}{2}(v + v_0)t$$

$$t = \frac{x - x_0}{\frac{1}{2}(v + v_0)}$$

$$t = \frac{3 \times 10^{-2}\text{ m}}{\frac{1}{2}(6 \times 10^{6}\text{ m/s} + 9 \times 10^{3}\text{ m/s})}$$

Because our initial velocity is almost a thousandth less than our final velocity, it's not going to account for much in the sum of the two, so we can just discard it. As always on the MCAT, numerical answer choices are spaced far apart in their values. The test encourages mental math and simplifying steps.

$$t = \frac{3 \times 10^{-2}\text{ m}}{\frac{1}{2}(6 \times 10^{6}\text{ m/s} + 0)}$$

$$t = \frac{3 \times 10^{-2}\text{ m}}{3 \times 10^{6}\text{ m/s}} = 1 \times 10^{-8}\text{ s}$$

16. B

It's generally a safe assumption that an MCAT problem is easier than it looks. Having internalized a few basic ideas will let this one come clear quickly. Firstly, we know that the acceleration is the rate of change of the velocity. An average rate of change over a duration of time maps onto a specific amount of change. We can determine the change in the velocity by plugging the given time values into the expression for velocity as a function of time. From there it's easy to determine the average rate of change over an interval of three seconds.

$$\boldsymbol{v}(t) = \frac{1}{3}\left[(t-1)^{(t+1)}\right]$$

$$\boldsymbol{v}(1) = \frac{1}{3}\left[(1-1)^{(1+1)}\right] = 0\text{ m/s}$$

$$\boldsymbol{v}(4) = \frac{1}{3}\left[(4-1)^{(4+1)}\right] = \frac{1}{3}\;3^{5} = 81\text{ m/s}$$

$$\bar{a} = \frac{v - v_0}{\Delta t}$$

$$\bar{a} = \frac{81\text{ m/s} - 0\text{ m/s}}{3\text{ s}} = 27\text{ m/s}^2$$

17. A

If you know both the initial speed and the angle at which it was launched you know both the magnitude and direction of the initial velocity. The only force on the projectile during flight is gravity which acts downward. Therefore, the vertical component of this vector will determine the time interval for the projectile to reach the peak. How much time will it take gravitational acceleration to deplete the initial vertical velocity to zero?

$$t_{\text{peak}} = \frac{v_{y_0}}{g}$$

The time to land will be twice this value.

$$t_{\text{range}} = \frac{2v_{y_0}}{g}$$

While it's in flight, the horizontal component of the velocity keeps plugging away at a constant value. The horizontal velocity takes the time interval afforded by the vertical velocity and moves the projectile down the field.

$$\text{range} = \frac{2v_{y_0}}{g}\, v_{x_0}$$

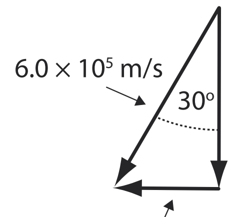
In other words, as long as there is no air friction and the ground is level, the range is completely determined by the initial velocity.

18. A

The negative charge experiences a uniform acceleration perpendicular to the plates towards the positive plate. The motion is directly analogous to projectile motionmotion, so the path will be a parabola.

19. D

Just as with projectile motion, we need to resolve the velocity of the electron to separate out the component which will affected by the acceleration, here due to the uniform electric field instead of gravitation.



$(6.0 \times 10^5\ \text{m/s})(\sin 30°)$
$= 3.0 \times 10^5\ \text{m/s}$
This is the velocity component affected by the electric field.

As with projectile motion, to answer the question of time in flight, we first need to ask how long it takes the electron to reach the peak. The time in flight will be twice that value.

$$t_{\text{peak}} = \frac{3.0 \times 10^5\ \text{m/s}}{1.5 \times 10^{12}\ \text{m/s}^2}$$

$$= 2.0 \times 10^{-7}\ \text{s}$$

$$t_{\text{range}} = 2\ t_{\text{peak}} = 4.0 \times 10^{-7}\ \text{s}$$

20. A

Solving this depends on the previous question, something you'll almost never see on the MCAT, so the question stem provides the time in flight, given as 4.0×10^{-7} s. The question stem asks for the minimum plate separation to prevent the particle from striking the far plate. This is another way of asking what is the distance from the positive plate of the peak of the electron's path. We use half that value, the time to reach the peak, 2.0×10^{-7} s, in the displacement equation. Note acceleration as negative because its direction opposes the initial velocity component.

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

$$x - x_0 = (3.0 \times 10^5\ \text{m/s})(2.0 \times 10^{-7}\ \text{s}) + \tfrac{1}{2}(-1.5 \times 10^{-12}\ \text{m/s}^2)(2.0 \times 10^{-7}\ \text{s})^2$$

$$= 3.0 \times 10^{-2}\ \text{m} = 3.0\ \text{cm}$$

Newton's Laws Practice Items

1. A moving body with no net forces acting upon it will...

A. come to rest.
B. exist in static equilibrium.
C. continue moving with constant velocity.
D. undergo a constant positive acceleration.

2. A block is at rest on an inclined plane. As the angle of incline is increased beyond the point at which the block begins to slide...

A. the normal force upon the block decreases.
B. the force of friction increases.
C. the force required to prevent sliding of the block decreases.
D. the weight of the block decreases.

3. A man (mass 150 kg) is standing on a scale in an elevator which is dropping free fall. According to the scale, how much does he weigh? $g = 10\ m/s^2$

A. 0 N
B. 15 N
C. 150 N
D. 1500 N

4. The force compelling an object to remain in uniform circular motion is called

A. the centripetal force
B. the reaction force
C. the centrifugal force
D. the equatorial force

5. A man is standing on a scale in an elevator which is accelerating downwards at 6 m/s^2. The scale registers 400 N. How much would the man normally weigh?

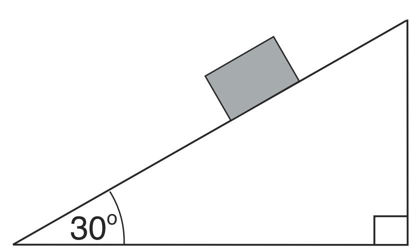
A. 100 N
B. 150 N
C. 800 N
D. 1000 N

6. A 2 kg object travels at 5 m/s in a certain direction. An unknown force is applied to the object along that same direction. Ten seconds of constant application results in the object traveling at the speed of 25 m/s. What is the magnitude of the unknown force?

A. 1 N
B. 2.5 N
C. 4 N
D. 10 N

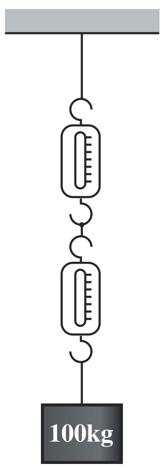
7. Which of the following combinations of materials for the block and plane below would prevent the block from freely sliding under the influence of gravity?

A. wood on wood (ms = .3)
B. steel on concrete (ms = .9)
C. steel on steel (ms = .7)
D. more than one of the above are correct



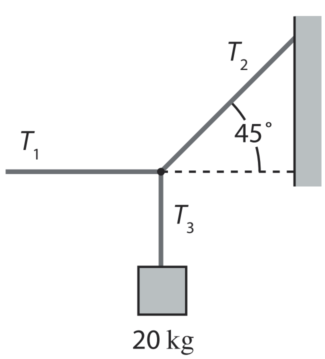
8. A 100kg mass is suspended from two spring scales connected together as shown in the figure at right. The scales are of negligible weight. In this arrangement

A. each scale reads 50N.
B. each scale reads 500N.
C. each scale reads 1000N.
D. the top scale reads 1000N and the bottom reads zero.



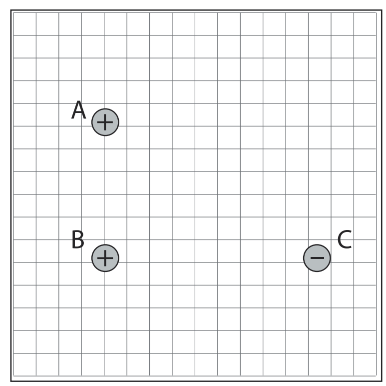
9. The weight of a 20kg mass is supported as shown in the figure below. Three cords of negligible weight are joined at a knot. What is the magnitude of tension T_1?

A. 14N
B. 20N
C. 200N
D. 280N



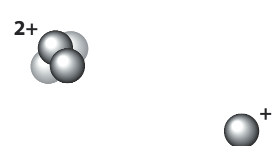
10. The units of electric field in the SI system may be referred to as either volts per meter or, equivalently, as newtons per coulomb. At the location of particle B depicted in the figure below, the electric field of particle A has a magnitude of 3.0×10^{-5} V/m, and the electric field of particle C has a magnitude of 4.0×10^{-5} V/m. Particle B possesses a charge of 1.5×10^{-6} C. What is the magnitude of the net force on particle B ?

A. 1.1×10^{-11} N
B. 1.4×10^{-11} N
C. 5.0×10^{-11} N
D. 7.5×10^{-11} N



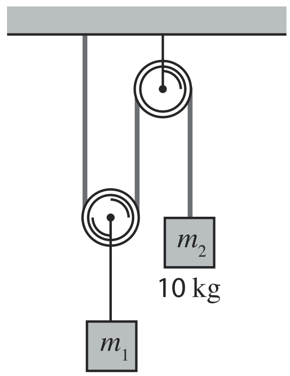
11. A proton (mass = 1.67×10^{-27} kg), moving at a speed of 3.0×10^{5} m/s in the vicinity of an alpha particle (mass = 6.64×10^{-27} kg) exerts a force of 6.7×10^{-19} N on the alpha particle. What force does the alpha particle exert on the proton?

A. 6.7×10^{-19} N
B. 1.3×10^{-18} N
C. 2.7×10^{-18} N
D. cannot be determined without also knowing the speed of the alpha particle



12. Neglecting friction and the mass of the pulley, what is the acceleration of the mass m_2 in the apparatus below?

A. $0\ m/s^2$
B. $5\ m/s^2$ in the downward direction
C. $5\ m/s^2$ in the upward direction
D. $10\ m/s^2$ in the downward direction



13. Traveling on a flat roadway an automobile speeds around a curve of radius R. The coefficient of kinetic friction between the car's tires and the roadway is μ_k, and the coefficient of static friction is μ_s. Which best expresses the maximum speed v_{max} at which the car can travel without slipping?

A. $\sqrt{\mu_s g R}$

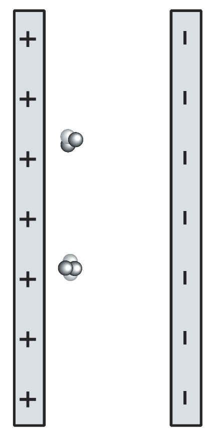
B. $\sqrt{\frac{\mu_k N R}{m}}$

C. $\frac{\mu_s N R}{m}$

D. $\frac{\mu_k N R}{m}$

14. Within an apparatus kept at near absolute zero temperature, an alpha particle, ${}^4_2\mathbf{He}^{2+}$, and an ionized helium-3 nucleus, ${}^3_2\mathbf{He}^{2+}$, begin moving at the same moment in time from a position near the surface of the positive plate in the parallel plate capacitor shown below. Which particle strikes the far plate first?

A. ${}^3_2\mathbf{He}^{2+}$
B. ${}^4_2\mathbf{He}^{2+}$
C. They will both strike at the same time.
D. Neither particle begins to move.



Newton's Laws
Answers and Explanations

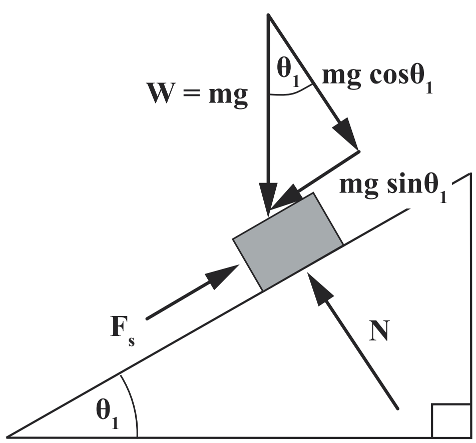
1. C

This is Newton's First Law. An object at rest stays at rest or an object in motion stays in motion with the same constant velocity unless acted upon by a net force.

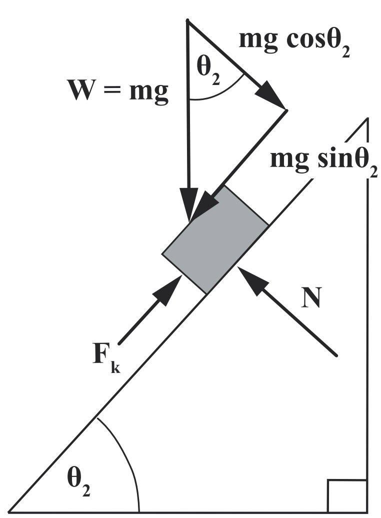
2. A

Let's look at the change in the free body diagram for the block on the inclined plane when we increase the angle of incline and it starts sliding.

Here is the block before the angle of incline is increased:



Here it is afterwards. (Note that static friction force has been replaced with kinetic friction force.)



The normal force, **N**, must be equal and opposite to the component of the block's weight perpendicular to the plane. As the angle of incline is increased, the component of the weight perpendicular to the plane decreases and so does the normal force.

3. A

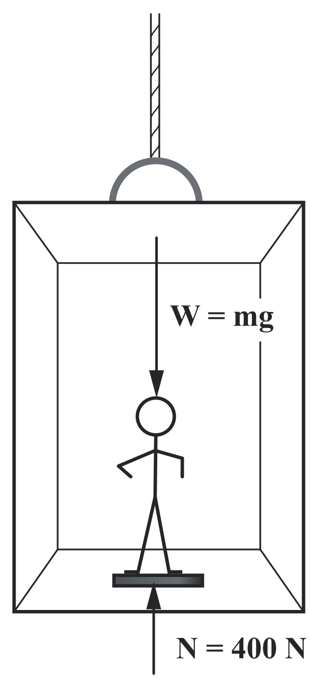
The men is undergoing acceleration due to gravity. For this to be the case, his weight is the only force acting on him. There is no normal force, so the scale registers 0 N.

4. A

In uniform circular motion, a centripetal force is the force that makes a body follow the circular path. Its direction is always perpendicular to the motion of the body and towards the center of the circle.

5. D

With this type of problem, it's a good idea to start with a free body diagram and feel your way.



Above is a free body diagram corresponding for the man on the elevator. We can derive an expression for the net force on the man from this free body diagram and use Newton's 2nd Law to relate this net force to his acceleration. The acceleration the man undergoes is the result of this net force acting on his mass. (Note we are calling the upward direction negative because it feels clearer mentally. It's okay to do it either way as long as you're consistent.):

$$\sum F = ma$$

$$mg - 400\text{N} = m\,(6\ \text{m/s}^2)$$

$$m(10\ \text{m/s}^2) - m\,(6\ \text{m/s}^2) = 400\text{N}$$

$$m(4\ \text{m/s}^2) = 400\text{N}$$

$$m = 100\text{kg}$$

$$mg = 1000\text{N}$$

6. C

Through Newton's 2nd Law, we know that to determine the force on the object we need to compute the acceleration it is undergoing. The acceleration is the rate of change of the velocity:

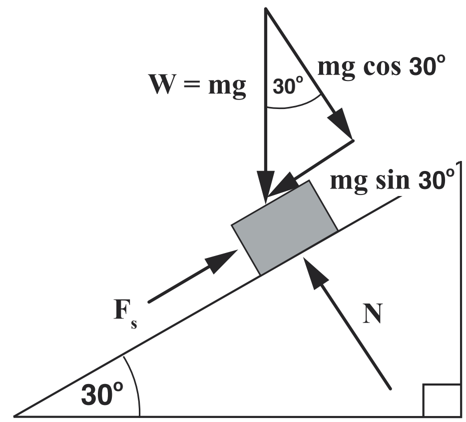
$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{\Delta t}$$

$$= \frac{25\ \text{m/s} - 5\ \text{m/s}}{10\ \text{s}} = 2\ \text{m/s}^2$$

Then applying the 2nd Law:

7. D

The block will not side if the component of the weight of the block down the plane is less than the threshold of static friction. If that case a static friction force equal and opposite to the component of the weight down the plane will prevent sliding.



The threshold of static friction equals the product of the coefficient of static friction and the normal force. The static friction force which occurs will be less than or equal to this value.

$$F_s \le \mu_s N$$

The normal force on the inclined plane is equal in magnitude to the component of the weight perpendicular to the plane, so the threshold of static friction can be expressed:

$$F_s \le \mu_s mg \cos 30^\circ$$

The block won't slide if the weight parallel to the plane is met by an equal and opposite static friction force.

$$mg \sin 30^\circ \le \mu_s mg \cos 30^\circ$$

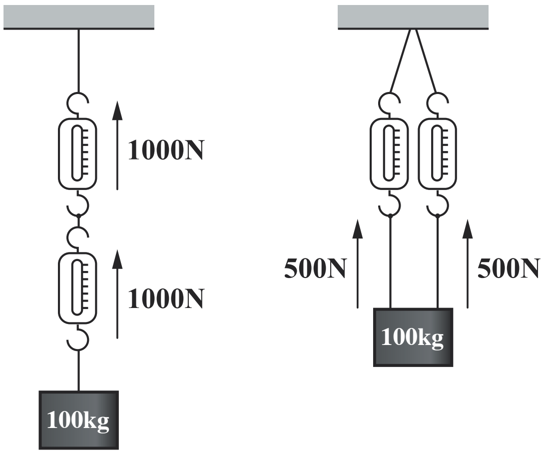
$$0.5 \le \mu_s 0.87$$

$$\mu_s \ge 0.6$$

Both steel on steel and steel on concrete satisfy this condition. With those materials, the block won't slide.

8. C

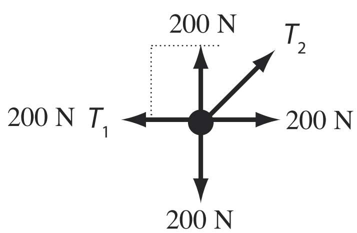
If the two spring scales were sharing the load, they would each show approximately half the weight. But if you hang one below the other, then each feels the full load.



9. C

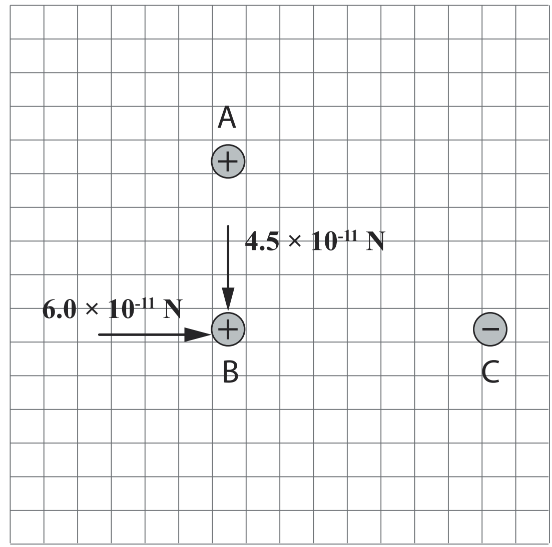
Solving cord tension problems nearly always begins with construction of a free body diagram. Focusing on where the cords meet, we know that, because net force equals zero, the cord(s) must supply an upward force equal and opposite to the weight.

This upward force is supplied solely by the vertical component of T_2, 200 N. And because the angle of T_2 is 45°, its horizontal and vertical components are equal. If the horizontal component of T_2 also equals 200 N, then the value of T_2 is also 200 N.

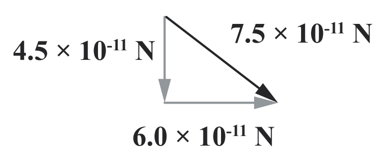


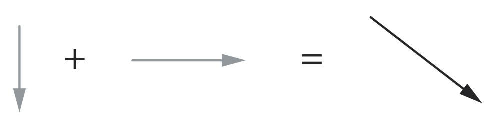
10. D

The question stem provides a bit of help in mentioning that the strength of an electric field can equivalently be thought of in terms of newtons per coulomb or volts per meter. Even though the given information is expressed in volts per meter, straightforwardly thinking about what newtons per coulomb must mean for an electric field leads you to the basic electrostatics formula $\boldsymbol{F = Eq}$. In other words, we simply multiply the field strengths of charges A and C at the location of charge B by the charge in coulombs of B to obtain the respective forces each exerts on B shown in the free body diagram below.



Now we perform vector addition to obtain the net force on B. The ratio of the forces are 3:4 and they are at a right angles, so vector addition will conform to the simple 3:4:5 right triangle ratio. A clue was provided in that the electric field strengths in the question stem have scientific notation coefficients 3 and 4.





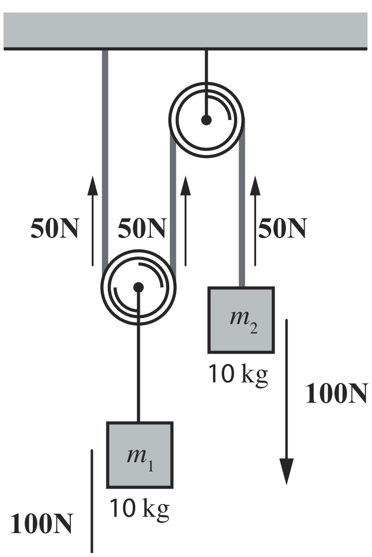
11. A

MCAT passages and question stems will often have a lot of hand-waving with scientific details. It can make an easy question seem a lot more complicated than it is. Always start with the assumption that the question is easy. In this case, this is simply Newton's 3rd Law. If an object A exerts a force on object B, then object B must exert a force of equal magnitude and opposite direction back on object A.

12. B

Sometimes there's a period of uncertainty the moment you first encounter a physics or chemistry problem. It can be like staring into the void, and you don't exactly know what to do. This is natural. It doesn't mean you're in trouble. Be patient and let your mind accommodate to the problem. Play with it a little bit. Be open to your intuition. Write down the given information. Some kinds of problems are convenient in that there are some very useful things to do while your mind adjusts to the problem. In a chemistry problem, you write molarity as moles per liter or molecular weight as grams per mole, for example. You're doing something useful and playing for a little time. Trust yourself. There's no reason to feel stress. Soon the problem will be unfolding, and then you see it.

In this type of problem, a very useful thing to do at the start is construct a free body diagram.



The tension in the rope is 50N to hold up the 10kg mass on the left. This translates as a net upward force of 50N on the mass on the right. Therefore, there is a net downward force of 50N on the 10kg mass. From Newton's 2nd Law, **F = ma**, we know then it is undergoing a 5 m/s^2 acceleration downward.

13. A

An object traveling on a curved path with a certain radius of curvature will experience a centripetal force equal to

$$F_r = \frac{mv^2}{r}$$

The threshold of static friction represents the maximum capacity of static friction for two surfaces. If a motion producing force exceeds this value, then sliding will occur. The threshold depends on the coefficient of static friction for the materials and the normal force.

$$F_s \leq \mu_s N$$

At the maximum speed, the centripetal force will have come to equal the threshold of static friction.

$$\frac{mv_{max}^2}{R} = \mu_s N$$

$$v_{max}^2 = \frac{\mu_s NR}{m}$$

The normal force between the car and the roadway will be equal in magnitude to the car's weight, *mg*, so we can simply the expression:

$$N = mg$$

$$v_{max}^2 = \mu_s g R$$

$$v_{max} = \sqrt{\mu_s g R}$$

14. A

We have two positively charged particles with the same magnitude of charge within the same uniform electric field. They both experience the same electrostatic force, $F = Eq$. However, because the helium-3 nucleus has a smaller mass, ie. 3 amu vs 4 amu for the alpha particle, it has less inertia. By Newton's 2nd Law, $F = ma$, the same force on a smaller mass will result in a greater acceleration, so the helim-3 nucleus will strike first.

Work & Energy Practice Items

1. The translational kinetic energy of a moving body at a given instant depends on . . .

- **A.** its weight
- **B.** its heat content
- **C.** the forces acting on it
- **D.** its speed

2. With regard to the work performed by the centripetal force upon an object undergoing uniform circular motion

- **A.** the centripetal force performs no work upon the object.
- **B.** the amount of work performed by the centripetal force in one revolution equals the magnitude of the force times the circumference of the circle.
- **C.** the amount of work performed by the centripetal force equals the change in the kinetic energy of the object.
- **D.** more than one of the above is correct.

3. A 5-kg brick falls from a scaffolding. Ignoring the effects of air friction, what is its kinetic energy 2 seconds later?

- **A.** 10 J
- **B.** 100 J
- **C.** 250 J
- **D.** 1000 J

4. A crane lifts a 1000-kg automobile 10 meters off the ground. Discounting friction, how much work is performed?

- **A.** 10,000 J
- **B.** 15,000 J
- **C.** 100,000 J
- **D.** 150,000 J

5. Even if there were no drag from the air, a pilot should know that the amount of jet-fuel required to bring about an acceleration from 150 to 300 m/s is

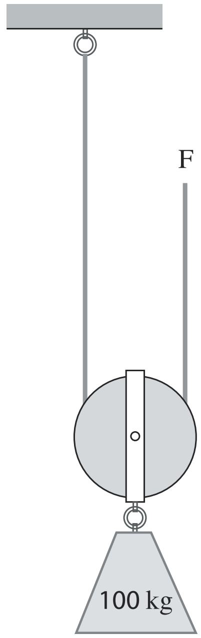
- **A.** half the amount required to accelerate from 75 to 225 m/s.
- **B.** equal to the amount required to accelerate from 75 to 225 m/s divided by the square root of 2.
- **C.** equal to the amount required to accelerate from 75 to 225 m/s.
- **D.** greater than the amount required to accelerate from 75 to 225 m/s..

6. Which of the following illustrates conservation of mechanical energy?

- **A.** A gun recoils when fired.
- **B.** An iceberg floats with only one-tenth of its volume above water.
- **C.** In sliding to a stop, the friction of an automobile's tires heat up the road.
- **D.** In a vacuum chamber, the bob of a perfectly frictionless pendulum is released and returns to exactly the same point.

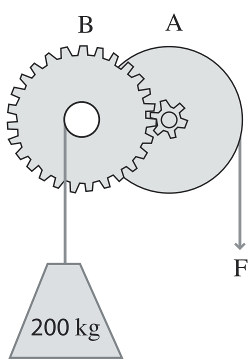
7. What is the mechanical advantage of the pulley system pictured below?

A. 1
B. 2
C. 3
D. 4



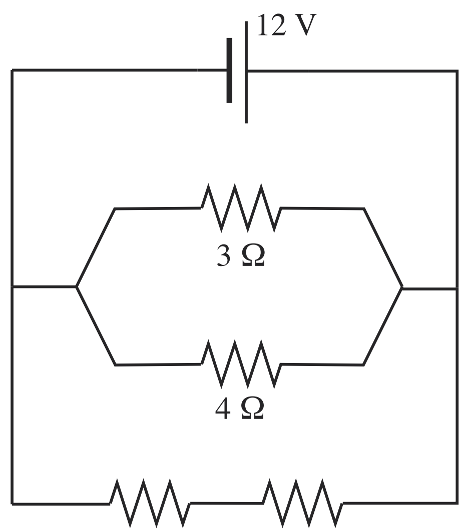
8. In the gear train below the radii of gear A and belt pulley is 20cm and the shaft of gear B and belt pulley is 5cm. Gear A has 6 teeth and gear B has 30 teeth. What minimum force **F** must be exerted to lift the mass of 200-kg?

A. 100N
B. 400N
C. 500N
D. 2000N



9. In traveling through the circuit depicted in the diagram below, a positive charge has three alternative pathways of movement from the positive pole to the negative pole of the voltage source. The charge could move either through the 3Ω resistor, the 4Ω resistor, or through the combination of 1Ω and 2Ω resistors. Which pathway represents the greatest decrease in electrostatic potential energy for the charge?

A. through the 3Ω resistor
B. through the 4Ω resistor
C. through the combination of the 1Ω and 2Ω resistors
D. the electrostatic potential energy decrease is the same for the three pathways

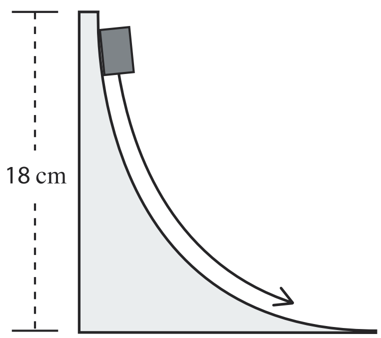


10. All of the following statements are correct concerning work *except*...

A. Work can be expressed as the product of the force times the component of displacement in the direction of the force
B. A kinetic friction force performs no work.
C. If performed by conservative forces, the amount of work does not depend on the path taken to reach a certain state.
D. The work performed equals the product of the power at which a mechanical system is operating and the time.

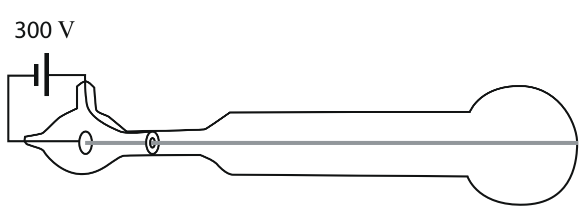
11. A block is released from rest to slide down the frictionless surface as pictured below. The block achieves a final speed of

A. 0.6 m/s
B. 1.9 m/s
C. 3.6 m/s
D. 6.0 m/s

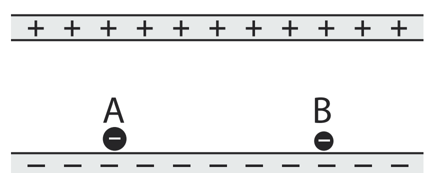


12. What is the electron speed in the cathode ray tube shown below? (Electron mass = 9.11×10^{-31} kg; 1 eV = 1.60×10^{-19} J)

A. 1.6×10^5 m/s
B. 9.1×10^5 m/s
C. 2.5×10^6 m/s
D. 1.1×10^7 m/s



13. As shown in the figure below, two charged plates containing uniform distributions of opposite charge are separated by a distance much smaller than the size of the plates. Two negatively charged particles are released from a position near to the negative plate into the space between the two plates. The two particles possess equal charge, but particle A is 4 times as massive as particle B. Particle B is observed to strike the positive plate first moving at speed v_B, followed later by particle A moving at speed v_A. Which of the following describes the relationship between the speeds v_A and v_B?



A. $v_A = 0.25\, v_B$

B. $v_A = 0.50\, v_B$

C. $v_A = 0.71 v_B$

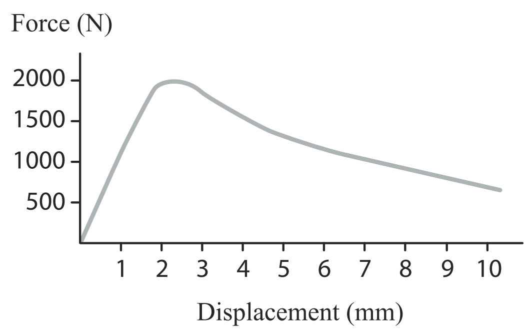
D. $v_A = v_B$

14. A 100 kg stone block slides down a ramp of height 10 meters. At the end of the slide, which lasts 4 seconds, the block's speed is 8 m/s. What is the average power delivered by the friction force of the ramp during the slide?

A. 320 W
B. 1250 W
C. 1700 W
D. 3200 W

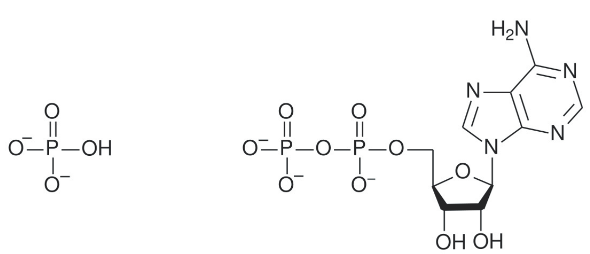
15. A dynamic hip screw (DHS) is a type of orthopedic implant designed for fixation of certain types of hip fractures in which the femoral head component is allowed to move along one plane. The figure below shows the compressive force vs. displacement graph for a particular DHS. Which of the following most closely approximates the amount of energy required to displace the femoral head component of this DHS a distance of 1 cm?

A. 1 J
B. 4 J
C. 12 J
D. 20 J



16. When a hydrogen phosphate molecule moves nearer to a molecule of ADP within an aqueous solution environment, this represents

A. an electrostatic potential energy increase
B. mechanical work by actin and myosin
C. a decrease in internal energy
D. heat flow

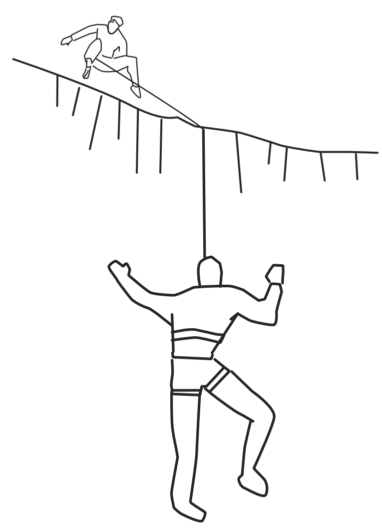


17. To overcome rolling friction and pull rail cars of mass 4.0×10^6 kg at a constant velocity of 36 km/hr a locomotive provides 3000 kW of power. What force does the locomotive exert on the rail cars?

A. 0 N
B. 1.4×10^4 N
C. 8.3×10^4 N
D. 3.0×10^5 N

18. To belay in rock climbing is to pull the rope in as another climber ascends below so that the climber will be safe. In the figure below, a 120kg man is belaying on level ground far back from the cliff-edge for a 100kg man hanging in mid-air below. The belaying man above begins sliding towards the cliff-edge, though doesn't fall over the edge. His friend drops 15m to the ground while he's sliding. The coefficient of kinetic friction between the belayer and the cliff surface is 0.8. At what approximate speed does his friend strike the ground below?

A. 7.7 m/s
B. 10.0 m/s
C. 12.2 m/s
D. 15.3 m/s



19. The only chemical elements that form stable two-atom homonuclear molecules at standard temperature and pressure (STP) are hydrogen (H_2), nitrogen (N_2) and oxygen (O_2), plus the halogens fluorine (F_2) and chlorine (Cl_2). In the gaseous state these molecules possess kinetic energy at the particle level, ie. thermal energy, in a variety of partitions corresponding to

A. vibration along the bond axis
B. rotation
C. translational motion
D. all of the above

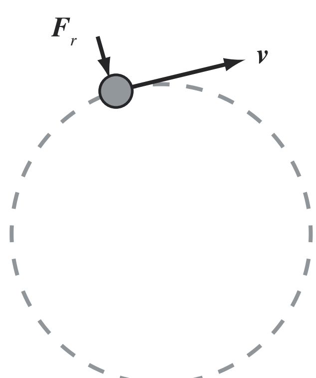
Work & Energy
Answers and Explanations

1. D
Anything that is moving has kinetic energy. The amount of kinetic energy in a moving body depends directly on its mass and speed.

$$K = \frac{1}{2}mv^2$$

Kinetic energy is the work invested in the motion of the body. It also equals the work required to bring it to rest.

2. A
The work performed equals the product of the magnitude of the force component parallel to the displacement and the magnitude of the displacement. In other words, the force needs to at least have a component parallel to the direction the motion to perform work. In uniform circular motion, however, the centripetal force is always perpendicular to the instantaneous displacement. The centripetal force performs no work.



3. D
Firstly, we need to determine the speed of the brick. The acceleration due to gravity is 10 m/s^2 (10 meters per second per second), so after 2s its speed will be 20 m/s.

$$\Delta v = a\,\Delta t$$

$$20 \text{ m/s} = (10 \text{ m/s}^2)(2 \text{ s})$$

Now that we know the speed, we can compute the kinetic energy.

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(5 \text{ kg})(20 \text{ m/s})^2 = 1000 \text{ J}$$

4. C
The work performed equals the product of the magnitude of the force component parallel to the displacement and the magnitude of the displacement, or, as long as you remember that only the component of the force parallel to the direction of the motion is performing work, then it's okay to say that the 'work equals force times distance.'

The force required to lift the car is the weight of the car ($W = mg$), so the work done is the product of the weight of the car and the height lifted.

$$W = (1000 \text{ kg})(10 \text{ m/s}^2)(10 \text{ m}) = 100{,}000 \text{ J}$$

Note that this is also the potential energy the car now possesses. Work is the vehicle for the transformation of energy.

5. D
The amount of jet-fuel consumed will be proportional to the amount of work done by the engines. The work-energy theorem states that the total work done on an object equals the change in its kinetic energy.

$$W = K_f - K_i = \Delta K$$

The kinetic energy of the jet is proportional the square of its speed.

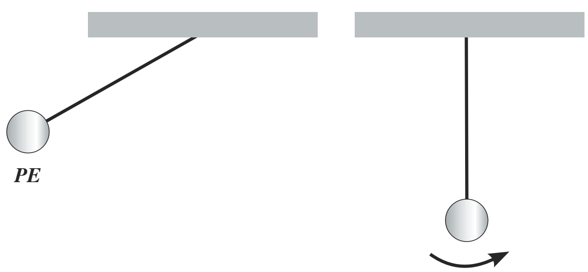
$$K = \frac{1}{2}mv^2$$

Because the speed is squared, the same magnitude of change in speed will mean a greater change in kinetic energy at higher speed. For example, the difference between 1 and 3 is the same as the difference between 5 and 7. The difference is 2 in both cases. However, the difference in squares are $3^2 - 1^2 = 8$ versus $7^2 - 5^2 = 24$.

6. D

The recoil of the gun demonstrates Newton's 3rd Law. The floating iceberg illustrates force equilibrium. The example of sliding friction demonstrates dissipation of mechanical energy into thermal energy.

The frictionless pendulum is a model system for the illustration of the conservation of mechanical energy.

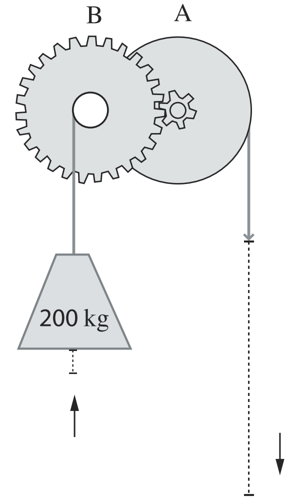


7. B

The purpose of a simple machine is to change the magnitude or direction of an applied force. The central concept in understanding machines is that the work output cannot exceed the work input. For a frictionless system, they are equal: $(\text{force}_{out})(\text{distance}_{out}) = (\text{force}_{in})(\text{distance}_{in})$. The mechanical advantage of a simple machine reflects its force multiplying effect, the ratio of the output force to the input force, by changing the distribution of force and distance constituting the work. In a frictionless system, where no work energy is lost to dissipation, the mechanical advantage also equals the ratio of input distance to output distance. The pulley system has a ratio of input distance to output distance of 2:1. In this pulley system, the input force travels two times the distance in performing the same work as the output force, so it can be two times smaller. The input force is multiplied. To lift the weight, a force only one half as great needs to be input.

8. A

The mechanical advantage of a simple machine reflects its force multiplying effect, the ratio of the output force to the input force, by changing the distribution of force and distance constituting the work. frictionless system, they are equal: $(\text{force}_{out})(\text{distance}_{out}) = (\text{force}_{in})(\text{distance}_{in})$. In other words, to determine the mechanical advantage, the ratio of force_{out} to force_{in}, we need to determine the ratio of the distances.



Let's feel our way with conversion factors. We know that one tooth of gear B moves forward after contacting one tooth of gear A, so we can use this to determine the ratio of revolutions of gear B to gear A.

$$\frac{1}{6}\frac{\text{revolution A}}{\text{tooth}} \quad \text{and} \quad \frac{1}{30}\frac{\text{revolution B}}{\text{tooth}}$$

$$\left(\frac{6}{1}\frac{\text{tooth}}{\text{revolution A}}\right)\left(\frac{1}{30}\frac{\text{revolution B}}{\text{tooth}}\right) = \frac{1\text{ revolution B}}{5\text{ revolution A}}$$

The belt advances a circumference per revolution for each gear. We use this to determine the distance ratio.

$$\left(\frac{1\text{ revolution B}}{5\text{ revolution A}}\right)\left(\frac{\text{revolution A}}{(2\pi)(20\text{cm})}\right)\left(\frac{(2\pi)(5\text{cm})}{\text{revolution B}}\right) = \frac{1}{20}$$

$(\text{force}_{out})(\text{distance}_{out}) = (\text{force}_{in})(\text{distance}_{in})$ so the mechanical advantage of this gear train is 20. The weight of 200kg is 2000N (mg). Lifting this weight using our gear train, therefore, requires an input force of 100N.

9. D

Like gravitational force, the electrostatic force is conservative. The change in potential energy doesn't depend on the path between two points.

10. B

Kinetic friction forces definitely do perform work. For an object sliding to rest on a surface, the work the kinetic friction force performs represents a transduction of mechanical energy into dissipative forms such as sound and thermal energy. For example, the product of the kinetic friction force for an automobile entering a sliding stop times the distance of the skid-marks equals the work performed by kinetic friction in bringing the automobile to rest. This will equal the kinetic energy of the car before the driver slammed on the brakes.

11. A

We solve this problem using the principle of the conservation of mechanical energy. The block's potential energy at the top of the slide will be fully converted into kinetic energy at the bottom.

$$\tfrac{1}{2}mv^2 = mgh$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2(10\text{m/s}^2)(1.8 \times 10^{-2}\text{m})}$$

$$v = \sqrt{3.6 \times 10^{-1}\text{ m}^2/\text{s}^2)}$$

Taking the square root of a number in scientific notation is much easier if the exponent of the base is an even number. It won't change the value of the number if we divide the base portion by 10 and multiply the coefficient by 10 but it makes computing the square root a simple problem.

$$v = \sqrt{36 \times 10^{-2}\text{ m}^2/\text{s}^2)}$$

$$v = 6 \times 10^{-1}\text{ m/s}$$

12. D

The electron volt is a unit of energy. An electron volt is the amount of work a 1 volt potential will perform on an elementary charge (the magnitude of charge of a proton or electron). 3000V will perform 300 eV of work on a single electron as it accelerates it within the electron gun of the cathode ray tube. The amount of work the electric field performs within the electron gun will equal the kinetic energy of the electron when it exits, so it exits the aperture of the electron gun with 300 eV of kinetic energy.

$$(3 \times 10^2\text{ eV})(1.6 \times 10^{-19}\text{ J/eV}) = 4.8 \times 10^{-17}\text{ J}$$

$$\tfrac{1}{2}mv^2 = 4.8 \times 10^{-17}\text{ J}$$

$$\tfrac{1}{2}(9.1 \times 10^{-31}\text{ kg})\, v^2 = 4.8 \times 10^{-17}\text{ J}$$

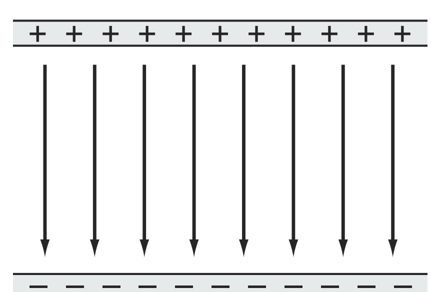
Mental math is a skill the MCAT openly encourages and rewards. Answer choices to quantitative problems are almost always widely spaced. The exam gives you a lot of latitude with mental math. You can see the step of dividing 4.5 into 4.8 as an invitation to simplify the quotient to 1.

$$v \sim \sqrt{1 \times 10^{14}\text{ m}^2/\text{s}^2}$$

$$v \sim 1 \times 10^7\text{ m/s}$$

13. D

There is a uniform electric field between the plates of a parallel plate capacitor.



Because the particles possess the same magnitude of negative electric charge, the upward force on each particle exerted by the electric field is the same.

$$F = Eq$$

An equal force pushing two particles through the same distance performs the same work (force times distance). This work is the vehicle for the transformation of electrostatic potential energy into kinetic energy.

$$W = K_f - K_i = \Delta K$$

In other words, although Particle B strikes first at a greater speed, each particle strikes the far plate with the same kinetic energy. If two particles possess the same kinetic energy, their speeds will be inversely proportional to the square roots of their masses.

$$\frac{1}{2}m_A\overline{v}_A^2 = \frac{1}{2}m_B\overline{v}_B^2$$

$$\frac{\overline{v}_A}{\overline{v}_B} = \frac{\sqrt{m_B}}{\sqrt{m_A}}$$

A particle which is four times more massive will be moving at half the speed.

14. A

Energy is neither created nor destroyed. The initial potential energy is transformed into a combination of kinetic energy and dissipative forms such as thermal energy and sound through the work of sliding friction.

$$PE_i = KE_f + F_k\Delta x$$

The initial potential energy:

$$PE_i = mgh = (100\text{ kg})(10\text{ m/s}^2)(10\text{ m}) = 10{,}000\text{ J}$$

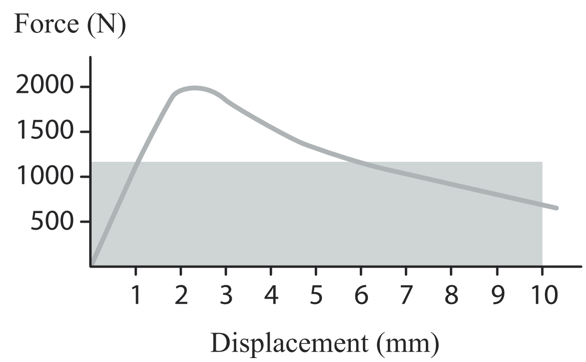
The final kinetic energy:

$$KE_f = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(100\text{ kg})(8\text{ m/s})^2 = 3{,}200\text{ J}$$

Therefore, 6,800 J were lost were lost due to the work of friction. This occurred over 4 s. This represents a rate of work performed, or power, of (6,800 J)/(4 s) = 1,700 W.

15. C

The stored energy equals the work done during compression. This work done equals the area under the force vs. displacement curve. If you find yourself in the position of needing to estimate the area under a curve, draw a best fit rectangle that balances what gets added to what gets left out as well as you can.



Using our rectangle as a guide, the area under the curve is approximately$(1200\text{ N})(1 \times 10^{-2}\text{ m}) = 12\text{ J}$.

16. A

Hydrogen phosphate has -2 total formal charge. ADP possesses a -3 charge. It would require work to move those two molecular ions closer together. When like charges are moved closer together, electrostatic potential energy increases.

17. D

'Work equals force times distance' (simplified when force and displacement are parallel). Fast objects cover greater distance in a given time, so more work is done by a force in that time; i.e. power is greater. The power associated with a force applied to a moving object is the product of the magnitude of the force and the object's speed.

$$P = Fv$$

The locomotive is providing 3000 kW of power to pull rail cars at 36 km/hr. Before using the equation above, we need to convert our units into SI system:

$$P = 3000\text{ kW} = 3 \times 10^6\text{ W}$$

$$v = \left(\frac{36\text{ km}}{\text{hr}}\right)\left(\frac{\text{hr}}{3600\text{ s}}\right)\left(\frac{1000\text{ m}}{\text{km}}\right) = 10\text{ m/s}$$

$$F = \frac{P}{v} = \frac{3 \times 10^6\text{ W}}{10\text{ m/s}} = 3 \times 10^5\text{ N}$$

18. A

Some of the potential energy of the climber below will be transformed into kinetic energy and some will be lost to the work of sliding friction.

$$PE_i = KE_f + F_k\Delta x$$

The initial potential energy:

$$PE_i = mgh = (100\text{ kg})(10\text{ m/s}^2)(15\text{ m}) = 15{,}000\text{ J}$$

To compute the work done by the man's sliding friction above the cliff-edge, we need to determine the kinetic friction force.

$$F_k = \mu_k N = \mu_k mg = (0.8)(100\text{ kg})(10\text{ m/s}^2)$$

$$F_k = 800\text{ N}$$

'Work equals force times distance' (simplified when force and displacement are parallel).

$$Work = (800\text{ N})(15\text{ m}) = 12{,}000\text{ J}$$

Therefore, the climber strikes the ground with 3000J kinetic energy.

$$KE_f = PE_i - F_k\Delta x$$

$$KE_f = 15{,}000\text{ J} - 12{,}000\text{ J} = 3000\text{ J}$$

Determining his speed from the kinetic energy:

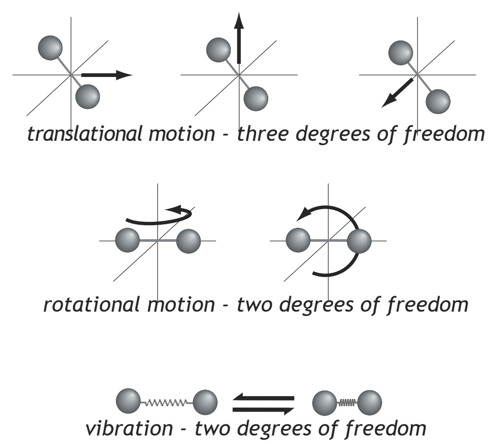
$$KE_f = ½mv^2$$

$$½\,(100\text{ kg})\,v^2 = 3000\text{ J}$$

$$v = 7.7\text{ m/s}$$

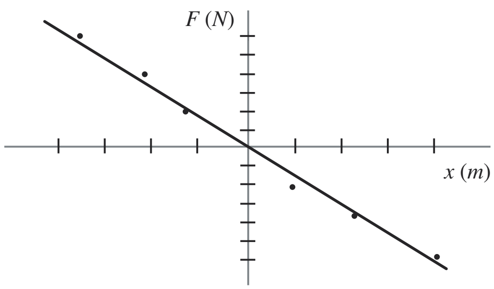
19. D

A monatomic gas molecule such as He possesses only kinetic energy deriving from its linear motion, but diatomic gas molecules such as H_2, O_2, etc. in addition to translational motion, can also rotate and vibrate. With the ability to manifest kinetic energy in both vibrational and rotational modes, a diatomic gas like has more partitions for thermal energy. Equipartition theorem predicts that as a sample of diatomic gas takes in heat, the energy spreads out into all seven of the degrees of freedom shown below. Diatomic gases can absorb heat flow into translational, rotational, and vibrational partitions. For this reason, the molar heat capacity of a diatomic gas will be greater than that of a monatomic gas.



Harmonic Motion Practice Items

1. After mass m is attached to a spring the value of the restoring force F is measured for different values of displacement from equilibrium. The graph below shows the value of the restoring force where x is the displacement of the mass from its equilibrium position.



What is the value of the spring constant k for this spring?

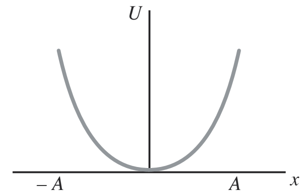
A. -1.5 N/m
B. -0.5 N/m
C. 0.5 N/m
D. 1.5 N/m

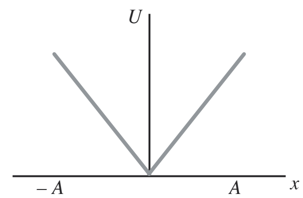
2. The frequency of a simple pendulum on Earth (for small oscillations) depends on the acceleration due to gravity and the length of the pendulum.

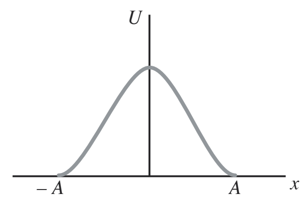
On the surface of the planet Mars, a simple pendulum 43 mm in length oscillates with a period of .65 s. What is the value of the acceleration due to gravity on Mars?

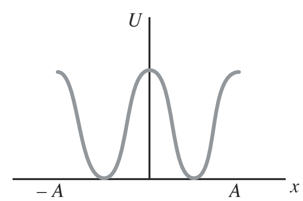
A. 0.67 m/s^2
B. 1.6 m/s^2
C. 2.7 m/s^2
D. 3.8 m/s^2

3. Which of the following graphs shows the elastic potential energy U as a function of displacement x for a simple mass-spring oscillator in the region between the amplitude positions, $x = \pm A$.

A.


B.


C.


D.


4. A particular mass-spring oscillator is not driven or damped. Which of the following would lead to a doubling of the frequency?

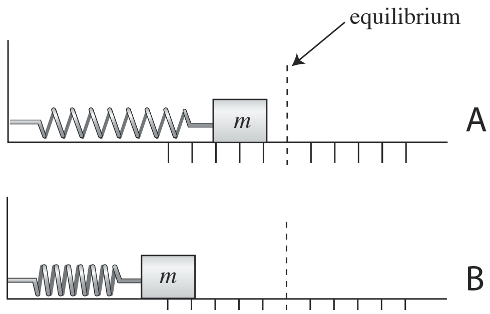
A. halving the mass on the spring
B. quadrupling the strength of the spring
C. doubling the strength of the spring
D. more than one of the above is true

5. Over the course of its oscillations the velocity of a mass-spring will be greatest when

A. the acceleration is greatest
B. the restoring force is zero
C. the displacement equals the amplitude
D. the restoring force is greatest

6. For the two identical springs, A and B, pictured below, displacement from equilibrium is measured as the distance the center of mass *m* has been moved from its equilibrium position. As can be seen, the two springs have different degrees of compression. The ratio between the two springs of stored potential energy equals

A. 1 : 16
B. 4 : 25
C. 2 : 5
D. 14 : 22



7. A frictionless mass-spring system undergoes simple harmonic motion with an amplitude of 10.0 cm. When the mass is 5.0 cm from its equilibrium position

A. 25% of the oscillator's energy is potential energy and 75% is kinetic energy
B. 50% of the oscillator's energy is potential energy and 50% is kinetic energy
C. 75% of the oscillator's energy is potential energy and 25% is kinetic energy
D. all of the oscillator's energy is potential energy

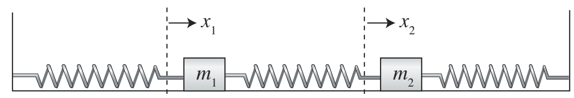
8. Which of the following changes would lead to an increase in the frequency of a simple, frictionless pendulum?

A. increasing the mass
B. decreasing the mass
C. increasing the length
D. decreasing the length

9. What strength of spring should be employed if the goal is to create a frictionless, horizontal mass spring-system that will oscillate a 100g mass at the same frequency as a simple pendulum that is 10 cm in length?

A. 0.1 N/m
B. 1.0 N/m
C. 10 N/m
D. 100 N/m

10. Two identical masses are free to slide over a frictionless horizontal surface. The masses are attached to one another, and to two immovable walls, by means of three identical light horizontal springs of spring constant k, as shown in the figure below. The extensions of the left, middle, and right springs are x_1, $x_2 - x_1$, and $-x_2$, respectively, assuming that $x_1 = x_2 = 0$ corresponds to the equilibrium configuration in which the springs are all unextended.



The acceleration of mass m_1 equals

A. $\dfrac{-kx_1 + k(x_2 - x_1)}{m_1}$

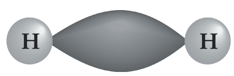
B. $\dfrac{-k(x_2 - x_1) - kx_2}{m_1}$

C. $\dfrac{kx_1 - kx_2}{m_1 + m_2}$

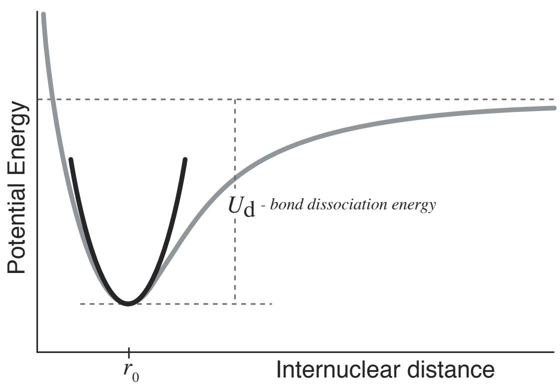
D. $\dfrac{kx_1 - kx_2}{(m_1 + m_2)/2}$

Passage (Questions 11-16)

The diatomic harmonic oscillator is a simple model for the vibration of chemical bonds. The model consists of two masses, m_1 and m_2, connected by a Hooke's Law spring with force constant k.



A chemical bond, if stretched too far, will break. A typical potential energy curve for a chemical bond as a function of r, the separation between the two nuclei in the bond is given in the figure below:



The bond dissociation energy curve pictured above (shown in gray in the figure) is given by a function of the form

$$U = U_d\left(1 - e^{-\alpha(r - r_0)}\right)$$

If the energy of the bond is not too high, then the potential energy curve is well approximated by a harmonic oscillator curve (shown in black in the figure). This curve is given by a simpler harmonic oscillator function

$$U = \frac{1}{2}k(r - r_0)^2$$

The exact quantum mechanical description of this model, obtained by inserting the above expression for the potential energy into the one-dimensional Schroedinger equation, yields an infinite set of allowed energies, with equal spacing of successive energy levels.

$$U_n = (n + \frac{1}{2})hf$$

n is a quantum number with discrete values of 0, 1, 2, etc. Planck's constant is denoted by h, and f is the frequency, which is determined by k and the two masses, present as their reduced mass, m_μ.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_\mu}}$$

$$m_\mu = \frac{m_1 m_2}{m_1 + m_2}$$

The reduced mass is a quantity which allows a two-body problem to be solved as if it were a one-body problem of mass m_μ with the position of one body with respect to the other as the unknown.

The harmonic oscillator is a useful model because its expression for the potential energy (as a parabola) is an adequate approximation to the true potential energy for many vibrations in real molecules, as long as one confines attention to that part of the potential energy surface close to the minimum. Important problems will arise if we probe the parts of this potential energy surface at much higher energies relative to the minimum.

11. According to information included in the passage the energy of a chemical bond in a highly excited state

A. can be usefully described using a mass-spring model
B. could result in bond dissociation
C. can be approximated as a parabola
D. increases as a function of the square of the internuclear distance

12. In the diatomic harmonic oscillator model of chemical bonds as energy level increases

A. vibrational frequency remains the same
B. potential energy asymptotically approaches the bond dissociation energy
C. the effective mass is reduced
D. molecular motion decreases

13. Which of the following could **not** be described with reasonable accuracy using the quantum mechanical model described in the passage?

A. the minimum photon energy to induce transition in vibrational energy in a covalent bond
B. the minimum photon energy required to break a chemical bond
C. the decrease in the frequency of bond vibration accompanying substitution of a heavier isotope of one of the bond atoms
D. the increase in vibrational frequency with bond strength

14. According to the model described in the passage substitution of two deuterium atoms for normal hydrogen atoms in a hydrogen molecule would result in the following change to the covalent bond?

A. an increase in the bond distance
B. no change in the ground state behavior
C. an increase in the strength of the bond
D. a decrease in frequency of the stretching vibration

15. The IR spectrograph of a particular organic molecule shows one type of C–C single bond absorbing infrared radiation of wavelength 10100nm (990cm^{-1}) and another bond, a C=C double bond, absorbing radiation at 6042nm (1645cm^{-1}). In terms of the diatomic harmonic oscillator model this difference can best be explained by which of the following?

A. the greater comparative vibrational frequency of C=C double bonds
B. the greater reduced mass of the C–C single bond
C. limitations in the predictive capability of the diatomic harmonic oscillator model
D. the greater comparative strength of C=C double bonds

16. For *k* and reduced masses of the size found in real molecules, the separation of adjacent energy levels, *hf*, is larger than thermal energy at standard temperature. Therefore, according to the model described in the passage,

A. at standard temperature, ground state atoms in chemical bonds are not vibrating.
B. the magnitude of the oscillation in the vibrational ground state is only a small fraction of U_d.
C. nearly all of the oscillators are in their ground state at standard temperature.
D. rotational energy levels of the diatomic molecule have higher wavelengths than the vibrational.

Harmonic Motion

Answers and Explanations

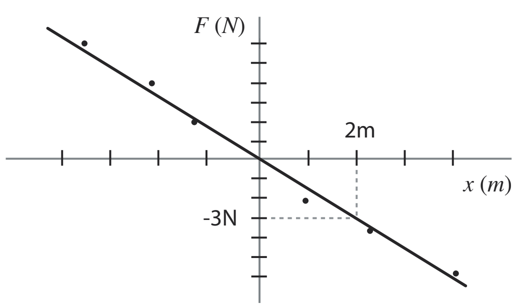
1. B

The restoring force supplied by a spring is proportional to the displacement from equilibrium, how far the spring has been stretched or compressed from the undeformed state. The negative sign in Hooke's Law accounts for the fact that the force exerted by the spring points in a direction opposite to the direction of deformation.

$$F = -kx$$

The proportionality constant, *k*, reflects the strength of the spring. A strong spring has a high *k* and responds to being stretched or compressed a short distance with a high restoring force. The units of *k* are N/m. In other words, *k* tells you how many Newtons of restoring force are supplied by the spring per meter of deformation.

To determine the spring constant for our spring, we can find a convenient point on the best fit line and use those values of displacement and restoring force. The value of 2m for displacement seems like a good place to look.



At 2m displacement, our best fit line predicts a restoring force of -3N.

$$F = -kx$$

$$-3\text{ N} = (-k)(2\text{ m})$$

$$k = 1.5\text{ N/m}$$

2. D

The frequency of a simple pendulum on Mars will depend on the acceleration due to gravity on Mars and the length of the pendulum.

$$f = \frac{1}{2\pi}\sqrt{\frac{g_{mars}}{L}}$$

First, we have a little work to do with the given information.

$$4\text{ cm} = 4 \times 10^{-2}\text{ m}$$

Also, we were given the period of the pendulum. We need to convert that to frequency.

$$f = \frac{1}{T} = \frac{1}{0.65\text{ s}} = 1.5\text{ s}^{-1}$$

Now we can compute g_{mars}.

$$\begin{aligned} g_{mars} &= (2\pi)^2 f^2 L \\ &= (6.28)^2(1.5\text{ s}^{-1})^2(4.3 \times 10^{-2}\text{ m}) \\ &= 3.8\text{ m/s}^2 \end{aligned}$$

Because the answer choices are fairly spread out numerically, you have latitude for back of the envelope calculations and mental math for the computation above. You just need to land reasonably close.

3. A

The greater the displacement the greater the restoring force ($F = -kx$). It takes more work to displace the spring a unit increment further from equilibrium. This is why the stored elastic potential energy in a mass-spring goes up with the square of the displacement from equilibrium. The graph is a parabola typical of quadratic functions.

$$U = \frac{1}{2}kx^2$$

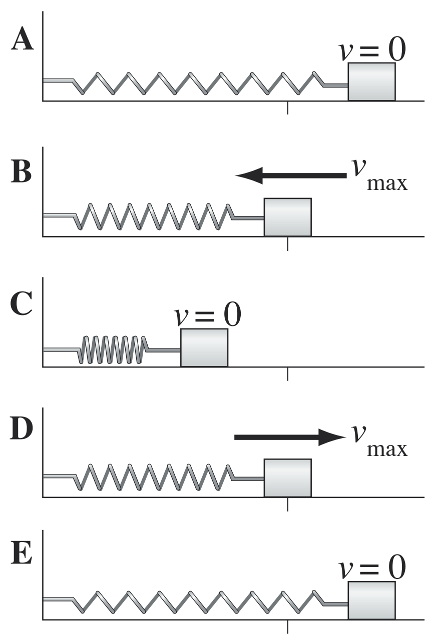
4. B

The frequency of a mass-spring is proportional to the square root of the spring constant.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

5. B

In the snap-shots from a single oscillatory cycle depicted below, the state of the mass-spring depicted in states **B** and **D** correspond to the description in the question stem. As the mass passes the equilibrium position, it is at maximum velocity. All of the energy in the system is in the form of kinetic energy.



6. B

Compression of spring A has displaced the center of its mass 2 units from equilibrium. Compression of spring B has displaced the center of its mass 5 units from equilibrium. The elastic potential energy stored in a spring is proportional to the square of the displacement from equilibrium.

$$U = \frac{1}{2}kx^2$$

Respective displacements of 2 vs. 5 units will correspond to a 4:25 ratio of stored energy.

7. A

The amplitude is the maximum displacement of the mass from its equilibrium position. At that maximum displacement, the mechanical energy of the spring is all the form of elastic potential energy. When moving past the equilibrium position, though, all of the energy is in the form of kinetic energy. For other displacements during oscillation energy apportions as a combination of potential and kinetic energy. Potential energy stored in a spring is proportional to the square of the displacement from equilibrium.

$$U = \frac{1}{2}kx^2$$

Therefore, respective displacements of 5 cm vs. 10 cm will correspond to a 25:100 ratio of stored potential energy. At the 5 cm position, the remaining 75% of total energy will be in the form of kinetic energy.

8. D

The frequency of a simple, frictionless pendulum depends only on its length. Shortening the length of a pendulum increases its frequency.

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

9. C

First we need to convert our units to SI system.

$$100 \text{ g} = 0.1 \text{ kg}$$
$$10 \text{ cm} = 0.1 \text{ m}$$

For the frequency of a mass-spring to equal the frequency of the pendulum, the following must be true:

$$\frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

$$\frac{k}{m} = \frac{g}{L}$$

$$k = \frac{mg}{L}$$

$$k = \frac{(0.1\text{ kg})(10\text{ m/s}^2)}{0.1\text{ m}}$$

$$k = 10\text{ N/m}$$

10. A

If you don't know what to do at first with a physics problem at first, try framing the problem as simply as you can. We're asked to find the acceleration of m_1. How do you determine the acceleration of a mass? You determine what forces are acting on it. What forces are being exerted on m_1? The force from the spring on the left and the force from the spring on the right. A spring responds to stretching or compressing by producing a restoring force.

$$F = -kx$$

For a displacement x_1 of the mass m_1 there will be a restoring force $-kx_1$ from the spring on the left. If the second mass m_2 were in a fixed position at $x_2 = 0$, the force from the spring on the right would be simply $-kx_2$, and that would be the net force combined with $-kx_1$. However, the degree of expansion or compression of the spring on the right may be affected by the displacement of m_2, so we need to adjust the force from the right in case the end of the spring is shifted x_2. The combination of the two forces can be expressed and resolved to show the acceleration as below.

$$F = -kx_1 + -k\,(x_1 - x_2)$$

$$F = -kx_1 + k\,(x_2 - x_1)$$

$$m_1 a = -kx_1 + k\,(x_2 - x_1)$$

$$a = \frac{-kx_1 + k\,(x_2 - x_1)}{m_1}$$

11. B

Like many MCAT passages, the passage combines fundamental concepts from undergraduate general science with concepts that are out of the scope of the expected foreknowledge for the test. Feel your way. Use what you know. Form a provisional understanding. Maintain open questions. How you handle yourself with out of scope material is a figure of merit for the exam.

This particular question combines reading comprehension with a basic sense of the covalent bond. In the passage we are told that the mass-spring approximation can only serve as a model of bond energy at low energies. At high energy, the curve of bond dissociation energy (the gray curve) can be seen approaching a maximum asymptotically. This maximum ($U = 0$; the minimum representing the two nuclei at the bond length is a negative value) represents the bond breaking. With high energy the nuclei can climb out of the well of electrostatic binding energy that is the bond.

12. A

The diatomic oscillator model involves treating the covalent bond as if it were a simple mass-spring system comprised of two masses connected by a spring.

The frequency of a mass-spring system does not depend on the amount of mechanical energy it possesses. Although increasing energy will increase the amplitude of the oscillation, the frequency depends on the spring constant and the mass. For a simple mass-spring, the frequency is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

The same principle applies when there are two masses. As described in the passage, where the formula below is given, the frequency only depends on the spring constant (analogous to bond strength) and the mass (reduced mass in the formula below).

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_\mu}}$$

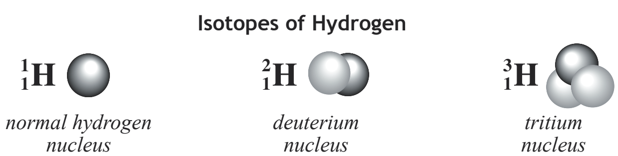
13. B

The quantum mechanical model described in the passage is a quantum mechanical model of the diatomic oscillator (mass-spring). The diatomic oscillator model breaks down at high energy. This can be seen in how the quantum mechanical energy formula below does not approach zero at high n, like the bond dissociation energy curve, but infinity, like the diatomic oscillator curve. It cannot model a bond breaking.

$$U_n = (n + \frac{1}{2})hf$$

14. D

Isotopes of an element have the same number of protons but a different number of neutrons in the nucleus, in other words, the same atomic number, but different neutron number, and, therefore, different mass number. Deuterium is the isotope of hydrogen with mass number 2.



In the diatomic oscillator model the vibrational frequency along the bond axis depends on the spring constant, k (bond strength), and the reduced mass of the two bonded atoms.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_\mu}}$$

$$m_\mu = \frac{m_1 m_2}{m_1 + m_2}$$

Substitution of an isotope would not change the strength of the bond, so k in our model would be unchanged. However, reduced mass, m_μ, does change. Normal hydrogen has a mass of 1 amu, so the two hydrogens together possess reduced mass = ½ amu, while deuterium has a mass of 2 amu, so reduced mass for two bonded deuterium atoms = 1. Increasing reduced mass will decrease the frequency of bond vibration.

15. D

Vibrational frequency along the bond axis depends on the spring constant, k (bond strength), and the reduced mass of the two bonded atoms.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_\mu}}$$

$$m_\mu = \frac{m_1 m_2}{m_1 + m_2}$$

A covalent bond will absorb infrared radiation with frequencies corresponding to its various modes of vibration. The C=C double bond absorbs at a higher frequency (shorter wavelength) than the C–C single bond because the frequency of its stretching vibration is greater. The reduced mass is the same in either case, so the increased frequency of vibration is due to greater bond strength, analogous to spring constant, k, in the diatomic oscillator model.

16. C

Thermal energy is the kinetic energy of the particles, ie. translational, rotational, and vibrational. The quantum mechanical model of the diatomic oscillator described in the passage presents a picture of bond vibration for a single molecule as a series of quantized states of increasing amplitude. Transitioning to a higher energy level in the quantum mechanical diatomic oscillator model requires more thermal energy than is present at standard temperature. That's what the question stem is saying. Only the most vigorous fraction, several standard deviations above the mean kinetic energy, have enough vibrational kinetic energy to do it, so nearly all the molecules are in the ground state. This question pitches hard, but it's right down the center of the plate.

Fluid Mechanics Practice Items

1. A bar of lead has the dimensions $2 \times 3 \times 5$ cm^3 and mass 330 gm. What is the specific gravity of lead?

A. 1
B. 10
C. 11
D. 50

2. If the bar of lead in problem #1 were submerged in water, what would be the apparent loss of weight on the bar?

A. 1/11 N
B. 0.3 N
C. 1 N
D. 300 N

3. In recreational scuba diving, the maximum depth limit is 40 meters. What is the pressure at that depth?

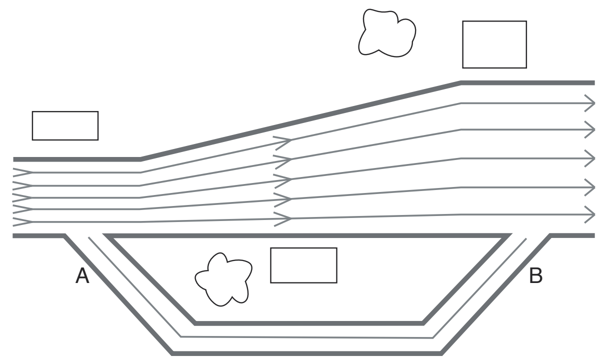
A. 4.0×10^3 Pa
B. 7.6×10^4 Pa
C. 4.0×10^5 Pa
D. 5.0×10^5 Pa

4. In laminar flow

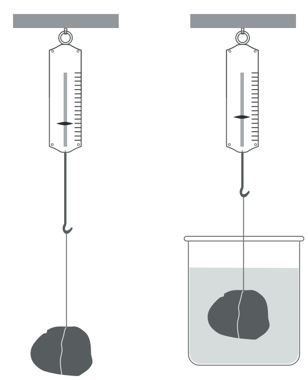
A. momentum is rapidly interchanged within the fluid.
B. the Reynolds number is greater than 2000.
C. eddy currents exist.
D. none of the above.

5. The figure below depicts the bird's eye view of a section of a canal system of uniform depth. The flow through the large canal moves left to right in a steady, streamline manner. The secondary flow in canal AB

A. moves left to right
B. moves right to left
C. does not move
D. must be turbulent



6. The weight of a sample of mineral ore was measured using a spring scale and found to equal 12.0 N suspended in air. Submerged in water the ore weighed 10.5 N. What is the specific gravity of the ore?

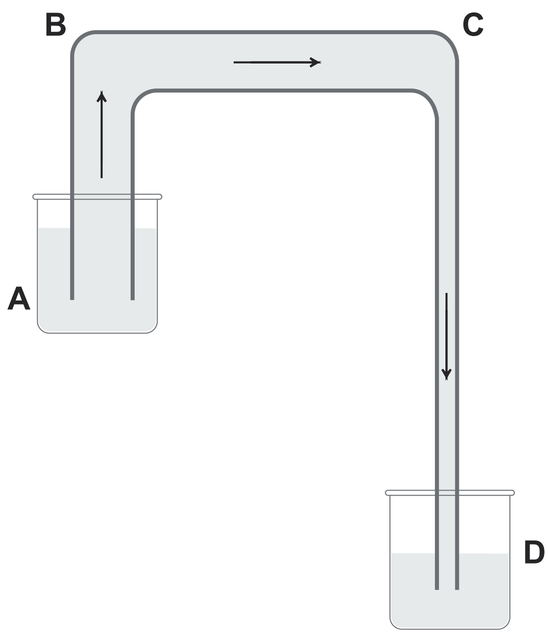


A. 1.1
B. 4.4
C. 8.0
D. 8.8

The following passage pertains to questions 7 - 10.

In the construction of a siphon, a tube is filled with liquid and the end of one arm of the tube is immersed in the liquid to be moved. Flow will occur up one arm of the siphon and then down the other. It is necessary in constructing a siphon for the arm drawing off the liquid to be shorter than the arm into which the liquid will flow, and that the end of the short arm of the siphon be kept at a level higher than the end of the long arm.

Flow through the siphon occurs because the column of liquid CD exerts more pressure at D than the column of liquid AB exerts at A.



7. If atmospheric pressure were to increase, the flow rate through the siphon above would

A. decrease
B. remain the same
C. increase
D. cease

8. The diameter of tube segment AB is 3 cm. The diameter of tube segment CD is 1 cm. When the flow speed through AB is 2 cm/s, what will the flow speed be through CD?

A. 6 cm/s
B. 9 cm/s
C. 15 cm/s
D. 18 cm/s

9. What is the approximate maximum height of segment CD in a functioning siphon in which the liquid is water?

A. ¾ m
B. 5 m
C. 10 m
D. 30 m

10. If instead of ordinary water, we were siphoning deuterium oxide (heavy water) which of the following differences would we observe in our system?

I. For a siphon of given dimensions, flow rate would decrease.
II. For a siphon of given dimensions, flow rate would increase.
III. The likelihood of turbulent flow would increase.

A. I only
B. II only
C. III only
D. both II and III

The following passage pertains to questions 11 - 16.

The dynamics of blood flow through an abnormal narrowing or stenosis in an artery can be approached at a fundamental level via Poiseuille's law and Bernoulli's principle. Poiseuille's law states a linear relationship between volumetric flow rate and pressure gradient in a real, Newtonian fluid

$$Q = \frac{\Delta P \pi r^4}{8 \eta l}$$

Q - volumetric flow rate
ΔP - pressure gradient
r - inner radius of the vessel
l - length of the vessel
η - blood viscosity

Bernoulli's principle describes how energy is conserved in the steady flow of an incompressible non-viscous fluid through interchange between kinetic energy, gravitational potential energy, and pressure along the flow-line.

$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

P - pressure
ρ - density
v - flow speed
g - acc. due to gravity
η - height

In a stenosed vessel, the more rapid flow of blood through a narrower lumen decreases the pressure gradient across the constriction. When pressure drops in any segment of the arterial system, it is due to both resistance from the stenosis and the conversion of potential into kinetic energy.

At some degree of narrowing, however, the affects of viscous dissipation outweigh continuity of volume flux in determining flow speed, and flow speed slows. Excess pressure can build up proximal to the stenosis until a critical stenosis is reached, defined as the percent stenosis at which intravascular flow approaches zero and the pressure approaches its maximum value. The critical stenosis is unique to vessel geometry and hemodynamics but has been shown to occur generally at approximately 80% to 85% obstruction of the major vessels in the human vasculature. At the point of critical stenosis, a sharp decrease of flow rate is observed as a result of the increased turbulence proximal to the stenosis.

If turbulence does not occur the pressure at the narrowed part will be substantially lower, which may result in caving in or even completely closing of the vessel. Then the flow velocity will slow down due to the frictional resistance, the kinetic energy will be converted to pressure and the vessel will reopen. This phenomenon will repeat itself causing the fluttering of the vessel.

Blood must be in continual motion to function properly. As blood flow is reduced through the stenosis, recirculation zones form distal to the stenosis, causing the flow to become stagnant. The stagnation of blood in these zones can trigger clotting mechanisms that lead to thrombosis. The mechanism by which blood clots as a result of reduced motion is termed stasis, and the resultant clot is the thrombus. It should be noted that the thrombus does not adhere strongly to the vessel wall and itself can be dislodged into the blood stream as an embolus and result in stroke.

11. A vascular surgeon carried out the graft of a section of the medial circumflex femoral artery substituting an autologous vessel of the same length but with twice the diameter. Assuming the pressure drop across the graft retains the same value it held prior to the surgery, how does flow speed through the graft compare to the old vessel?

A. 1/4 as great
B. the same
C. 4 times greater
D. 16 times greater

12. Based on the information contained in this passage, what is the best prediction in the change in the flow speed within a short stenosis of a large blood vessel after 30% narrowing has occurred. Turbulence does not occur and viscous dissipation is minimal?

A. flow speed approximately doubles
B. flow speed remains the same
C. flow speed decreases by 40%
D. flow speed decreases by 90%

13. As stenosis progresses to occlusion in a horizontal vessel, the pressure drop across the stenosis reaches 100% of the maximum and the flow rate is zero. At this stage

A. a thrombus will form distal to the stenosis.
B. turbulent flow will result distal to the stenosis.
C. the viscous dissipation of energy will be at a maximum.
D. the prestenotic pressure will equal the pressure at the origin of the parent vessel.

14. Which of the following, if present, reduces the descriptive relevance of **both** Poisseuile's law and Bernoulli's principle?

A. turbulent flow
B. viscous dissipation
C. vessel narrowing
D. a Newtonian fluid

15. An aneurysm is caused by the weakening of the arterial wall where a bulge occurs and the cross-section of a vessel increases considerably. At the cross-section of an aneurysm

A. flow velocity will be reduced and the pressure will be reduced.
B. flow velocity will be reduced and the pressure will increase.
C. flow velocity will increase and the pressure will be reduced.
D. flow velocity will increase and the pressure will increase.

16. If blood were less viscous

A. turbulent flow would occur less often in the circulatory system.
B. systolic blood pressure would increase.
C. pressure before a stenosis would more closely equal pressure after the stenosis.
D. separation between systolic and diastolic blood pressure would increase.

Fluid Mechanics
Answers and Explanations

1. C

The specific gravity of a body is the ratio of its density to the density of water. It's convenient to keep our given information in cgs units because we know the density of water in those units.

$$\rho_{H_2O} = \frac{1\text{ g}}{\text{cm}^3}$$

Density equals the mass per unit volume, so the density of our lead sample is:

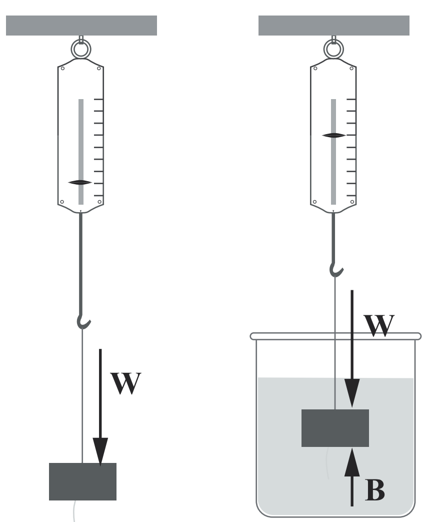
$$V = (2\text{ cm})(3\text{ cm})(5\text{ cm}) = 30\text{cm}^3$$

$$\rho_{Pb} = \frac{330\text{ g}}{30\text{cm}^3} = \frac{11\text{ g}}{\text{cm}^3}$$

Our sample is eleven times more dense than water, so its specific gravity is 11.

2. B

When a physics problem uses the phrase 'the apparent loss of weight' with a submerged object, what is meant by that is the buoyant force.



Archimedes' principle states that the upward buoyant force that is exerted on a body immersed in a fluid, whether fully or partially submerged, is equal to the weight of the fluid that the body displaces.

$$B = W_{\text{fluid displaced}}$$

$$30\text{cm}^3\ H_2O = 30\text{g}\ H_2O$$

$$30\text{g} = .03\text{ kg}$$

$$B = W_{H_2O} = mg = (.03\text{ kg})(10\text{ m/s}^2) = 0.3\text{ N}$$

3. D

The pressure of a liquid open to the atmosphere is equal to the atmospheric pressure on the surface and increases with the depth by an amount equal to the product of the density of the liquid, the acceleration due to gravity, and the depth.

$$P = P_a + \rho gh$$

The atmospheric pressure in SI units is 101,000 Pa or 1×10^5 Pa. The SI density of water is 1000kg/m^3.

$$P = 1 \times 10^5\text{ Pa} + (1000\text{ kg/m}^3)(10\text{ m/s}^2)(40\text{ m})$$

$$P = 5 \times 10^5\text{ Pa}$$

4. D

A high Reynolds' number, rapid interchange of momentum in the fluid, and eddy currents are all characteristics of turbulent flow, not laminar flow, which is steady, streamline flow.

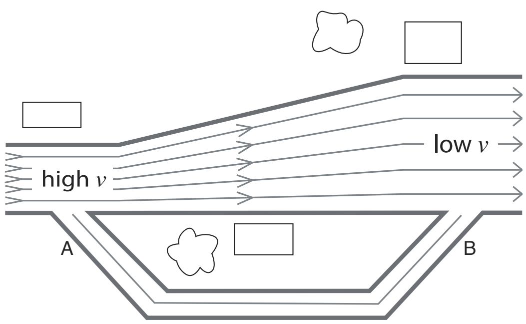
The Reynolds number is a dimensionless quantity empirically found to predict the onset of turbulence. Turbulence is highly probable if the Reynolds number is greater than 2000, and streamline flow is highly probable if the Reynolds number is less than 2000.

5. B

Because the volume of fluid entering one end of the main canal equals the volume leaving in a given time, the flow can be described by the continuity equation.

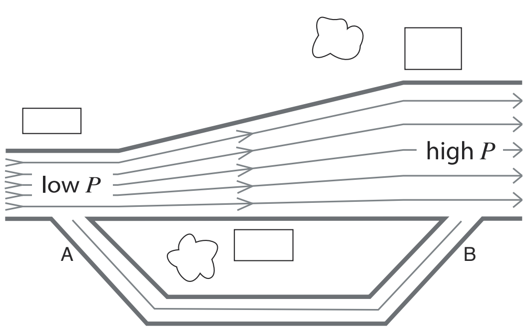
$$A_1 v_1 = A_2 v_2 = \text{constant}$$

As a consequence, the water flowing through the widened section, where the cross-sectional area is greater, is moving more slowly than through the narrow section.



Bernoulli's equation tells us that where the flow speed is slowest, the pressure is greater. (Because the grade is level, $\rho g y$ doesn't change).

$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

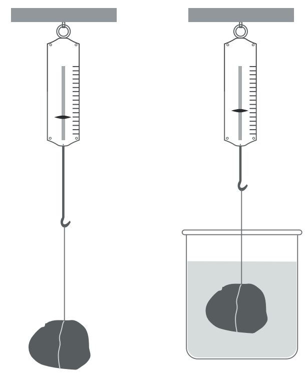


Because the pressure is higher at B than at A and water flows in the "reverse" direction in the secondary canal.

6. B

The spring scale measures a lower weight for the submerged ore because of the upward buoyant force exerted on it. The buoyant force is equal to the weight of the water that the ore displaces.

$$B = W_{\text{fluid displaced}}$$



A 1.5 N apparent loss of weight occurred. This apparent loss of weight is the buoyant force. How can we use this value to determine the specific gravity of the ore, the ratio of the density of the ore to the density of water?

The buoyant force is the weight of the water displaced, measured as an apparent loss of weight of the ore. A submerged object displaces a volume of water equal to its own particular volume. 1.5 N is the weight of this particular volume of water, and this is the same volume as the volume of the ore. Because for a given volume, the weight (mg) of a substance is proportional to its density (m/V), the ratio of the weight of the ore suspended in air to the buoyant force when it is submerged in water must equal its specific gravity. Eureka! The specific gravity is 12.0 N / 1.5 N = 8.0.

7. B

Bernoulli's equation provides a useful framework for understanding the simple siphon.

$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

If you compare the profile under Bernoulli's of the static fluid (the flow rate is very low in the reservoir itself) in the upper reservoir at y_{high} to the fluid exiting the pipe into the lower reservoir at y_{low}, there is a transfer of energy per volume element into flow speed driven by the difference in the $\rho g y$ terms. In other words, the atmospheric pressure is not a factor in driving the flow speed. (However, without atmospheric pressure setting a baseline the fluid could not establish a column to get over the high bend of the siphon.)

8. D

Because the volume flux into the siphon has to equal the volume flux exiting the syphon, continuity of volume flux will govern the relationship between the flow speeds.

$$A_1 v_1 = A_2 v_2 = \text{constant}$$

The flow speed is inversely proportional to cross-sectional area. The diameter of tube segment AB is 3 cm. The diameter of tube segment CD is 1 cm. Because the cross-sectional area is πr^2, the ratio of the cross sections of these tube segments is 9:1. The speed through AB will be nine times greater than the speed through CD, or 18 cm/s.

9. D

There is a limit to the height over which one can siphon a fluid. This occurs if the height of fluid in the lower of the two segments is such that the pressure changes with depth, ρgh, would track upwards from a reservoir at atmospheric pressure and predict negative pressure at the height of the siphon. The pressure in the tube cannot be negative. The fluid will come apart and fall down the two columns as air pressure no longer suffices to hold it up in the tube. We can determine the depth of a fluid over which the change in pressure equals atmospheric pressure:

$$\rho gh = 100{,}000 \text{ Pa}$$

$$(1000 \text{ kg/m}^3)(10 \text{ m/s}^2)h = 100{,}000 \text{ Pa}$$

$$h = 10 \text{ m}$$

10. C

Flow depends on the interchange of potential energy per unit volume in the upper reservoir to kinetic energy per unit volume exiting segment CD as described in Bernoulli's law. The sum of these three terms is constant anywhere in the flow line.

$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

Because density appears in both of the terms, alterations of density will not affect the flow speed in the lower reservoir. The same logic underlies the independence of mass in governing the speed of a falling body.

However, increasing density will increase Reynolds number. A high Reynolds number is a predictor of turbulent flow.

$$RN = \frac{\rho v d}{\eta}$$

RN = Reynolds number
ρ = fluid density
v = flow speed
d = geometrical property of the flow (diameter of obstruction, pipe width)
η = viscosity

11. C

Through Poiseuille's law we can predict that doubling vessel radius results in an sixteen fold increase in volume flux.

$$Q_{old} = \frac{\Delta P \pi r_{old}^4}{8 \eta l}$$

$$Q_{new} = \frac{\Delta P \pi (2r_{old})^4}{8 \eta l}$$

$$Q_{new} = 16 \frac{\Delta P \pi r_{old}^4}{8 \eta l} = 16\, Q_{old}$$

Volume flux, Q, measured in SI in cubic meters per second, is the rate of volume flow across a unit cross-sectional area in the vessel. Volume flux equals the product of cross-sectional area and flow speed.

$$Q = Av$$

To reiterate, the new volume flux is 16× greater.

$$Q_{\text{new}} = 16Q_{\text{old}}$$

We can express this relationship in terms of cross-sectional area and flow speed.

$$A_{\text{new}}v_{\text{new}} = 16A_{\text{old}}v_{\text{old}}$$

With twice the diameter, the new vessel has four times the cross sectional area.

$$A_{\text{new}} = 4A_{\text{old}}$$

New we can express the new flow speed in terms of the old.

$$4A_{\text{old}}v_{\text{new}} = 16A_{\text{old}}v_{\text{old}}$$

$$v_{\text{new}} = 4v_{\text{old}}$$

12. B

One of the challenges in fluid mechanics is knowing the proper framework to apply. When viscous dissipation is an important factor, such as occurs with flow of a viscous fluid through a narrow vessel,

Poiseuille's law applies. The question stem is careful to make the point that the stenosis has not measurably affecting the energy in the flow, so it does not affect the volume flux through the vessel as a whole. In that case, the problem is a straightforward continuity equation problem.

$$A_1v_1 = A_2v_2$$

Decreasing the vessel diameter by 30% will result in a radius 0.7 times the original, so the cross sectional area decreases to 0.49 the original value. The cross-sectional area is approximately half what it was, so the flow speed approximately doubles within the stenosis.

13. D

When blood flow ceases, you have a static fluid. The pressure is the same everywhere at the same depth within a static fluid.

14. A

Turbulent flow is fluid motion characterized by chaotic changes in pressure and flow velocity. It is in contrast to a laminar flow, which occurs when a fluid flows in parallel layers, with no disruption between those layers. Turbulence occurs when kinetic energy in the flow overcomes the damping effect of viscosity. Turbulent flows are always highly irregular. For this reason, turbulence problems are normally treated statistically rather than deterministically. Neither Poiseuille's law nor Bernoulli's equation can be used to model turbulent flow.

A Newtonian fluid refers is one that conforms to the simplest mathematical model of the viscosity (ratio of sheer stress to rate of change of sheer strain). While Bernoulli's equation will deviate the greater the effect of viscous dissipation in the flow of a Newtonian fluid, it is the model system for Poiseuille's law.

15. B

Continuity of volume flux tells us that as vessel diameter increases, flow speed decreases.

$$A_1v_1 = A_2v_2 = \text{constant}$$

Bernoulli's equation tells us that as flow speed decreases pressure increases.

$$P + \frac{1}{2}\rho v^2 + \rho gy = \text{constant}$$

16. C

The less the viscosity the less the viscous dissipation of energy along the flow due to peripheral resistance and friction between fluid laminae. You can see this in Poiseuille's law. Imagine viscosity, η, decreasing with a constant volume flux, Q. Do you see how $\Delta P/l$ will decrease? A lower viscosity would lead to a lower loss in pressure per unit length. The fluid is not losing as much energy.

$$Q = \frac{\Delta P \pi r^4}{8 \eta l}$$

Waves Practice Items

1. The wavelength of a harmonic wave divided by its speed of propagation is equal to

A. the frequency
B. the angular frequency
C. the wave number
D. the period

2. The greater the amplitude of a sound wave, the greater its

A. frequency
B. wavelength
C. wave number
D. energy

3. The speed of sound through air is approximately 343 m/s at normal room temperature. An E note produced by a piano has a frequency of 660 Hz. What is its wavelength?

A. 0.5 m
B. 2.0 m
C. 11 m
D. 20 m

4. Which of the following statements about sound wave is true?

A. They can pass through a vacuum.
B. Their speed does not depend upon the medium of propagation.
C. They travel as longitudinal waves.
D. They cannot be reflected.

5. Which of the following phenomena is **not** characteristic of sound waves:

A. reflection
B. polarization
C. diffraction
D. interference

6. Without changing the length of the string, the tension of a stretched string is increased 9 times. The fundamental frequency

A. increases 9 times
B. decreases 9 times
C. increases 3 times
D. remains the same

7. The fundamental frequencies of a set of banjo strings of different gauge but fixed length and tension are

A. the same
B. inversely proportional to string diameter
C. directly proportional to Young's modulus
D. distributed in simple integer ratios

8. Sound travels at a lower speed through air than through water. Which of the following is the best explanation for this?

A. Water is less dense than air.
B. Water is more dense than air.
C. Air is more compressible than water.
D. Water is more structured at the intermolecular level than air.

9. When two-out-of-tune flutes attempt to play the same note, one produces a tone that has a frequency of 392 Hz, while the other produces 406 Hz. When a tuning fork is sounded together with the 392 Hz tone, the combined notes quaver at a frequency of 9 Hz. When the same tuning fork is sounded together with the 406 Hz tone, a frequency of 5 Hz is produced. What is the frequency of the tuning fork?

A. 383 Hz
B. 397 Hz
C. 401 Hz
D. 411 Hz

10. During the day, the upper atmosphere is cooler than air at ground level, while during the night, the upper levels are warmer than the lower levels. Traveling more slowly through more dense media, the tendency during the day is for sound waves to veer upwards through the atmosphere, while at night the sound waves veer more downwards. To which of the following is this situation most analogous?

A. the refraction of light
B. convection currents in the ocean
C. the Doppler effect
D. the thermal equilibrium of radiation

11. An organ pipe with one closed end is 0.75 m long. The speed of sound in the room is 330 m/s. Which of the following notes does the pipe produce as its fundamental mode?

A. A (110 Hz)
B. A (220 Hz)
C. B (248 Hz)
D. E (660 Hz)

12. The trombone is the loudest instrument in the orchestra. A trombone produces a loudness of 110 dB from one meter away. Two trombones from one meter away will produce a loudness of

A. 113 dB
B. 120 dB
C. 130 dB
D. 220 dB

13. The sound level at a distance 5 m from a point sourcce is 100 dB. At what distance will the sound level be 80 dB?

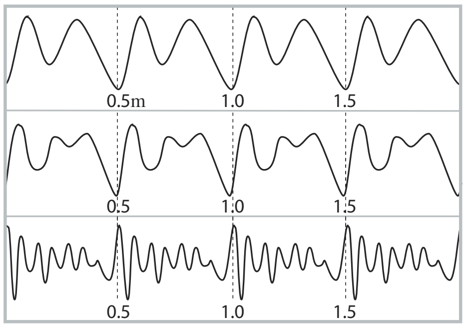
A. 20 m
B. 25 m
C. 50 m
D. 500 m

14. A low flying jet zooms past an airport tower. The speed of sound in the warm air of the airfield is 350 m/s. The pitch frequency of its engine noise shifts from 440 Hz from the listener's perspective as it is flying towards the tower to 330 Hz after it has passed. What is its approximate speed?

A. 50 m/s
B. 65 m/s
C. 70 m/s
D. 110 m/s

15. The three complex sound waves depicted in the figure below

A. have the same wavelength
B. are traveling through different media
C. possess different amplitudes
D. are transverse waves



Passage (Questions 16-21)

Seismic waves are waves of energy that travel through the Earth's layers, and are a result of an earthquake, explosion, or a volcano that imparts low-frequency acoustic energy. The propagation velocity of the waves depends on density and elasticity of the medium. Velocity tends to increase with depth, and ranges from approximately 2 to 8 km/s in the Earth's crust up to 13 km/s in the deep mantle.

There are two types of waves which travel within the body of the Earth. P waves are sometimes called compressional waves or primary waves or push-pull waves and they are propagated by movements of the material in the Earth parallel to the direction in which the wave is moving. S waves are also called shear waves or secondary waves and they propagate by movements of the Earth perpendicular to the direction in which the wave is moving.

P wave velocity depends on the bulk modulus, shear modulus and the density of the medium

$$v = \sqrt{\frac{\frac{4}{3}B + S}{\rho}}$$

S wave velocity depends on the shear modulus and density of the medium

$$v = \sqrt{\frac{S}{\rho}}$$

The crust mantle boundary was discovered in 1909 by a seismologist named Mohorovici (Yugoslav), as a result of his study of an earthquake in Croatia at that time. He found that, out to about 150 km, the time it took for the earthquake waves to reach each seismograph station was proportional to the distance the station was from the earthquake. He determined that the P wave velocity of the upper crust must be about 6 km/s. However, for stations greater than about 150 km from the earthquake, waves arrived with a higher average velocity.

Mohorovici calculated that the distance at which the change in velocity occurred (about 150 km) can be used to calculate the depth to velocity increase. He calculated that the depth to this velocity jump was about 30 km. We interpret this velocity jump as the crust-mantle boundary. At short distances, the "direct waves" that travel along the surface will arrive first. However, at greater distances, the P waves that travel down to the mantle, and are bent and travel along the top of the mantle at the higher velocity, can arrive before the waves traveling directly along the surface.

16. The velocity of seismic waves tends to increase with depth because

A. the medium is both more compressible and less dense at greater depth
B. the density and plasticity of the medium both increase with depth
C. elastic modulus and density both increase at greater depth
D. pressure and rigidity increase faster than density with depth

17. Which of the following may be observed with S waves but not with P waves?

A. polarization
B. constructive interference
C. travel through unsaturated sediments
D. amplitude

18. Within a geological medium through which both P waves and S waves are traveling, which seismic waves are faster?

A. P waves
B. S waves
C. It depends on the medium.
D. Both travel at the same speed.

19. When a seismic body wave encounters a lithological boundary layer, the incident ray can transform into several new rays. Some of the energy goes into the new layer but is bent, and some is reflected back up to the surface. The ray entering the new layer has undergone

A. wave splitting
B. refraction
C. polarization
D. interference

20. The higher velocity P waves observed by Mohorovici as described in the passage

A. had struck the crust mantle interface at the critical angle
B. had undergone conversion from S waves to P waves at the crust mantle interface
C. had been reflected by the crust mantle interface
D. had traveled a greater overall distance

21. From the S wave shadow zone observed with seismic readings taken on the opposite side of the Earth from an earthquake, it has been deduced that no S waves pass through the outer core. The most likely reason is that

A. The greater pressure at deeper levels opposes the perpendicular displacements of the S waves.
B. It is impossible to produce a shearing stress by displacing a section of liquid.
C. The S waves are reflected by the denser strata at that depth.
D. The S waves reach a threshold beyond which the gravitational force can no longer act to restore wave displacement.

Waves

Answers and Explanations

1. D

It's hard to resist answering frequency with this question. It can be this way with the easiest questions on the MCAT. Before toggling the answer, take a moment to check to make sure of what you think you see. It just takes just a moment. The MCAT is always testing your attention.

Frequency is not the wavelength divided by wave speed. That would be the reciprocal of frequency.

$$f = \frac{v}{\lambda}$$

The period is the reciprocal of the frequency. The period tells you how many seconds per cycle.

$$T = \frac{1}{f}$$

2. D

Amplitude is the magnitude of maximum disturbance of the medium, during one cycle of a periodic wave, known as peak deviation. The amplitude of a wave is related to the amount of energy it carries. A high amplitude wave carries a large amount of energy. We talk about the energy of sound in terms of intensity (W/m^2). Intensity goes up with the square of the amplitude of a sound wave.

3. A

The speed of a harmonic wave is the product of wavelength and frequency.

$$v = \lambda f$$

Divide frequency into wave speed to get the wavelength.

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{660 \text{ s}^{-1}} = 0.5 \text{ m}$$

4. D

Sound waves are longitudinal waves. Vibration is in the same direction as wave propagation.

5. B

Polarization is the phenomenon of transverse waves only. It does not happen in longitudinal waves such as sound waves. The disturbances in longitudinal waves are in the direction of wave propagation. In transverse waves the disturbances are perpendicular to the direction of wave propagation. Electromagnetic waves are transverse and may be polarized.

6. B

The wavelength of the fundamental vibrational mode of a stretched string is twice the length of the string. If the length of the stretched string isn't altered, the wavelength of the fundamental mode doesn't change. However, the speed of the wave on the string does change. The speed is proportional to the square root of the tension.

$$v = \sqrt{\frac{F}{\mu}}$$

Frequency equals wave speed divided by wavelength.

$$f = \frac{v}{\lambda}$$

A three fold increase in wave speed corresponds to a three fold increase in frequency.

7. B

The wavelength of the fundamental mode is twice the length of the string, so the wavelength is the same for all of the banjo strings. However, frequency is proportional to the wave speed and the wave speed is different.

$$f = \frac{v}{\lambda}$$

And wave speed is proportional to the square root of the tension and inversely proportional to the mass per unit length.

$$v = \sqrt{\frac{F}{\mu}}$$

In other words, the frequency is inversely proportional to the mass per unit length.

A banjo string is an extended cylinder. The mass per unit length is proportional to the volume per unit length. The volume per unit length equals the cross-sectional area. The cross sectional area is proportional to the square of the diameter.

In summary! The frequency is inversely proportional to the square root of the square of the diameter, or, more plainly, frequency is inversely proportional to the diameter.

8. C

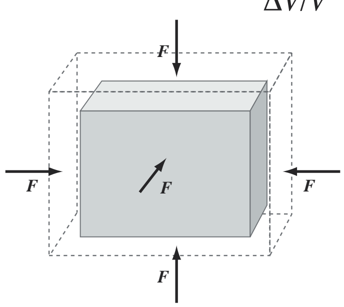
The speed of sound in a particular medium equals the square root of the bulk modulus of the medium divided by its density.

$$v = \sqrt{\frac{B}{\rho}}$$

Sound travels more slowly in air than water, but, by itself, the lower density of air vs. water predicts sound to travel faster in air. The reason sound travels more slowly in air is that air has a much lower bulk modulus.

The bulk modulus is a property of a material. Its the ratio of an applied pressure to the percent compression observed of the material.

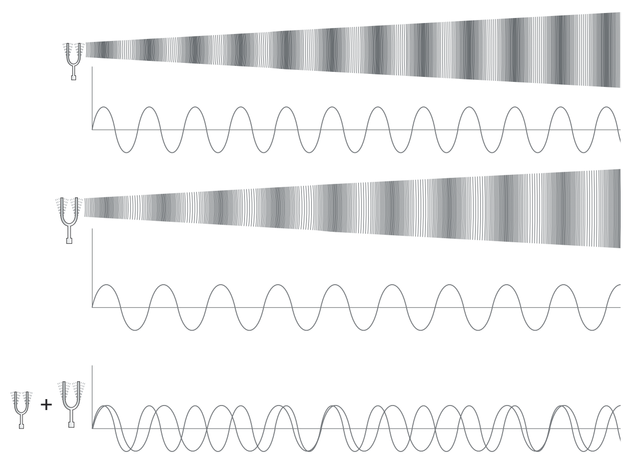
$$\text{bulk modulus} = -\frac{F/A}{\Delta V/V}$$



A high value of B indicates a material resists compression, while a low value indicates volume appreciably decreases under uniform pressure. The reciprocal of the bulk modulus is compressibility, so a substance with a low bulk modulus has high compressibility. Air has a much higher compressibility than water, in other words, a much lower bulk modulus. Hence, sound travels more slowly in air than water despite the lower density of air.

8. C

Beats are fluctuations in sound intensity produced when two tones nearly equal in frequency are sounded simultaneously. When sounds of slightly different frequencies are in the same position in space, the sound waves are periodically in phase, resulting in constructive interference, and then out of phase, resulting in destructive interference. The resulting audible pulsations in intensity are called beats.

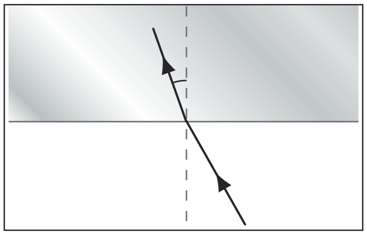


The number of beats one hears per second, the beat frequency, is simply the arithmetic difference between the frequencies of the contributing sound waves.

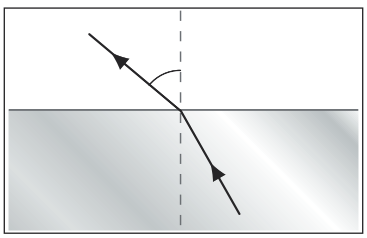
$$f_b = f_1 - f_2$$

Our tuning fork produces a 9 Hz beat frequency with the 392 Hz tone and a 5 Hz beat frequency with the 406 Hz tone. 9 Hz from 392 and 5 Hz from 406, its frequency must be 401 Hz.

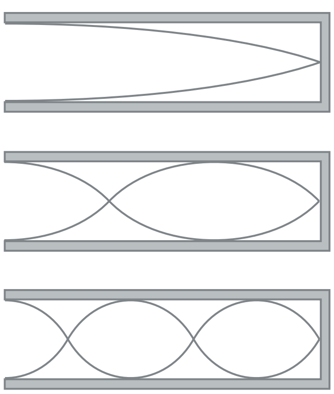
10. A
In the transmission of light, as light moves from a fast to a slow medium it bends towards the normal due to refraction.



As light is transmitted from a slow to a fast medium it bends away from the normal.



11. A
The wavelength of the fundamental mode of the standing waves in an air column closed at one end is four times the length of the pipe.



$$\lambda_n = 4L,\ \frac{4L}{3},\ \frac{4L}{5},\ \ldots\ \frac{4L}{n}$$
$$(n = 1,3,5,\ldots)$$

The wavelength of the fundamental mode is

$$\lambda_n = 4L = 4(0.75\text{ m}) = 3.0\text{ m}$$

To determine frequency when you have the wavelength, divide the wavelength into the wave speed.

$$f = \frac{v}{\lambda} = \frac{330\text{ m/s}}{3\text{ m}} = 110\text{ Hz}$$

12. A
Intensity measures the actual energy flux produced by a sound source in watts per square meter.

The loudness scale is related to intensity, but it is based on how humans perceive sound, and is proportional to the logarithm of intensity.

$$\beta = 10\log\left(\frac{I}{I_0}\right)$$

If I_1 is the original intensity, doubling the number of amplifiers will produce a new intensity of $2I_1$. The decibel level with this new intensity will be:

$$\beta_2 = 10\log\left(\frac{2I_1}{I_0}\right) = 10\left[\log\left(\frac{I_1}{I_0}\right) + \log 2\right]$$

Doubling the intensity results in the addition of 10 log (2) decibels, or about 3.

If you don't remember that the common logarithm of 2 is about 0.3, then ask yourself, to what power do I need to raise 10 to get 2? Well, 2 is a bit less than the cube root of 10, so the logarithm of 2 is a bit less than one third.

A faster path for answering this particular question would be to remember the following:

Increase intensity 10 fold and you add 10 dB.

Increase intensity 100 fold and you add 20 dB.

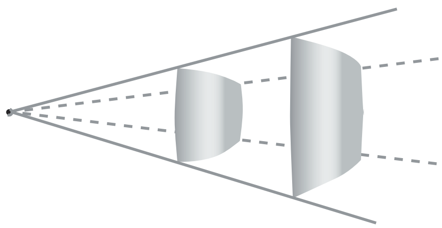
Increase intensity 1000 fold and you add 30 dB.

We only doubled intensity, so the addition to loudness is necessarily less than 10 dB. Choice 'A' is the only possibility.

13. A

Sound waves spread out into space from a point source. The spatial volume into which the sound waves are spreading increases and the intensity decreases with the square of the distance from the point source. The increasing volume of space into which the sound has spread dilutes its intensity.

$$I \circ \frac{1}{r^2}$$



The loudness scale is related to intensity, but it is based on how humans perceive sound, and is proportional to the logarithm of intensity.

$$\beta = 10 \log \left(\frac{I}{I_0}\right)$$

They tell us the loudness at a distance 5 m from a point source is 100 dB. To determine the distance at which the sound level would be 80 dB remember that a decrease in 20 dB represents a 100 fold decrease in intensity. To *decrease* intensity 100×, we need to *increase* the distance from the point source 10×. The sound level will be 80 dB at 50m.

14. A

The Doppler effect causes the frequency measured by an observer to be different than the frequency of the source whenever there is relative motion between the source and observer. A coefficient can be derived, depending on the speeds of the waves, observer, and source, to adjust the source frequency to the observer frequency.

$$f' = f\left(\frac{v \pm v_0}{v \mp v_s}\right)$$

f' = observed frequency
f = source frequency
v = wave speed
v_o = speed of observer
v_s = speed of source

If the observer is moving toward the source, use the positive sign in the numerator, which would lead to an increased observed frequency. If the observer is moving away from the source, use the negative sign. If the source is moving toward the observer, use the negative sign in the denominator, which would lead to an increased observed frequency. If the observer is moving away from the source, use the negative sign.

In our problem, the observer is stationary. The first equation below describes the shift in observed frequency as the plane approaches the tower, the second after it has passed.

$$440 \text{ Hz} = f\left(\frac{350 \text{ m/s}}{350 \text{ m/s} - v_s}\right)$$

$$330 \text{ Hz} = f\left(\frac{350 \text{ m/s}}{350 \text{ m/s} + v_s}\right)$$

At this stage we can foresee combining these two equations to solve for v_s.

$$440 \text{ Hz}\,(350 \text{ m/s} - v_s) = f\ (350 \text{ m/s})$$

$$330 \text{ Hz}\,(350 \text{ m/s} + v_s) = f\ (350 \text{ m/s})$$

It makes sense to hold off on doing any arithmetic. Combining the equations should simplify things for us.

$$(440 \text{ Hz})(350 \text{ m/s}) - (440 \text{ Hz})\,v_s = f\ (350 \text{ m/s})$$

$$(330 \text{ Hz})(350 \text{ m/s}) + (330 \text{ Hz})\,v_s = f\ (350 \text{ m/s})$$

We subtract the second equation above from the first and then solve for v_s.

$$(110 \text{ Hz})(350 \text{ m/s}) - (770 \text{ Hz})\,v_s = 0$$

$$350 \text{ m/s} - 7v_s = 0$$

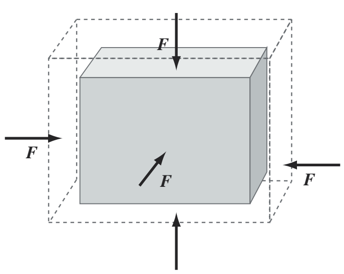
$$v_s = 50 \text{ m/s}$$

15. A
These are periodic but non-sinusoidal wave forms. With sinusoidal waves, the wavelength is the distance from crest to crest or trough to trough, but the true definition of the wavelength is the distance over which the wave's shape repeats. These all have the same wavelength.

16. D
For both P waves and S waves, wave speed is greater with greater elastic modulus. P waves are compression waves. Both shear and bulk modulus are relevant with P waves. S waves are transverse. The only relevant modulus with S waves is shear modulus.

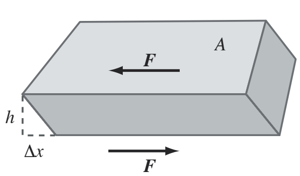
Bulk modulus the ratio of an applied pressure to the percentage compression observed of the material. The higher the bulk modulus the more rigid the material is to pressure.

$$\text{bulk modulus} = -\frac{F/A}{\Delta V/V}$$



The higher the shear modulus the more rigid the material is to a shearing stress.

$$\text{shear modulus} = \frac{F/A}{\Delta x/h}$$



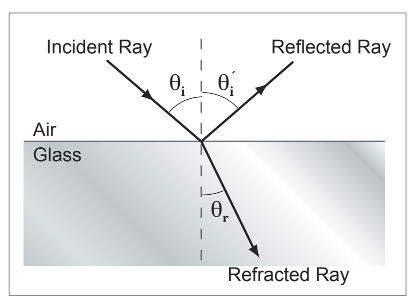
Whether P waves or S waves, the more rigid the medium the faster the wave. The more dense the material the slower the wave. Choice 'D' is the only answer which is logically consistent with this underlying basis for wave speed.

17. A
S waves are transverse. P waves are longitudinal. Only transverse waves can be polarized. With transverse waves, the disturbances are perpendicular to the direction of wave velocity. There can be variation in the plane of oscillation transverse to wave velocity. Polarization describes selection preference in this geometric orientation. With longitudinal waves on the other hand, the oscillations only occur along the wave parallel to wave velocity. It doesn't make sense to speak of "polarization" of longitudinal waves.

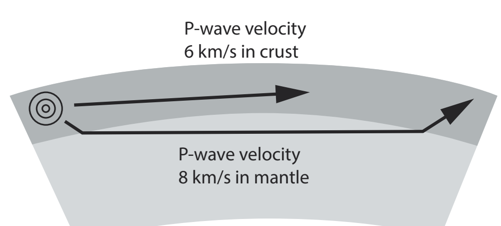
18. A
You can see this in the expressions for wave speed. By mathematical necessity, the expression for P wave speed on the left below must be greater than S wave speed on the right.

$$\sqrt{\frac{\frac{4}{3}B + S}{\rho}} > \sqrt{\frac{S}{\rho}}$$

19. B
The figure below shows a similar process of reflection and refraction of electromagnetic waves (light) at the boundary between air and glass.



20. D
The mental projection in reading comprehension should be along the lines of the figure below.



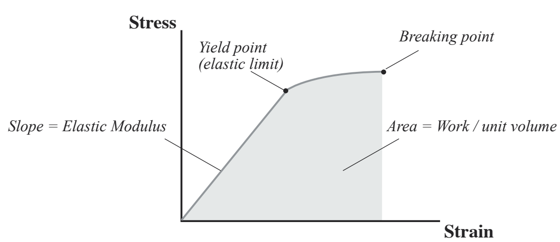
21. B

Let's get a little orientation to the topic of elasticity. It's not on the exam but it's extremely MCAT adjacent. However, it intersects with several important topics that are on the outline, for example, what determines the speed of sound in a particular medium. Elasticity is right on the edge of the scope of the test. The speed of sound in a particular medium equals the square root of the bulk modulus of the medium divided by its density. Bulk modulus is an elastic property of the medium which describes how hard the material is to compress.

$$v = \sqrt{\frac{B}{\rho}}$$

The elastic properties of a solid body govern the manner of its deformation under external forces. A stress is the array of forces causing a deformation. Strain measures the degree of deformation. The elastic modulus governs the relationship between stress and strain below the elastic limit, also called the yield point. After the yield point has been reached, the deformation is no longer completely reversible (the object no longer resumes its original shape when the stress is removed). If the elastic modulus for a particular material has a high value, objects made from that material are difficult to deform (and a reversible deformation stores much energy for a given strain).

$$\text{elastic modulus} = \frac{\text{stress}}{\text{strain}}$$



Curve of **Stress vs. Strain** *which describes the mechanical deformation of a solid body.*

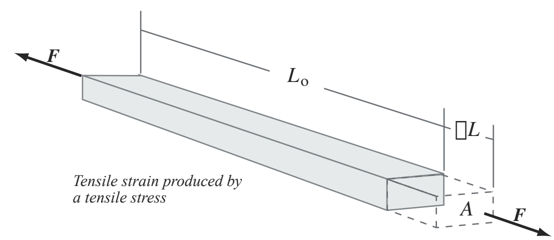
Three types of elastic modulus describe the three major kinds of deformation, Young's modulus (tensile stress and strain), shear modulus (shear stress and strain), and bulk modulus (volume stress and strain).

$$\text{Young's modulus} = \frac{F/A}{\Delta L/L_o}$$

tensile stress = $\frac{F}{A}$ tensile strain = $\frac{\Delta L}{L_0}$

F = force
A = cross-sectional area

ΔL = change in length
L_0 = original length



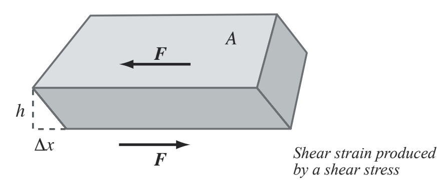
Tensile strain produced by a tensile stress

$$\text{shear modulus} = \frac{F/A}{\Delta x/h}$$

shear stress = $\frac{F}{A}$ shear strain = $\frac{\Delta x}{h}$

F = tangential force
A = area of face being sheared

Δx = horizontal distance sheared face moves
h = height of object



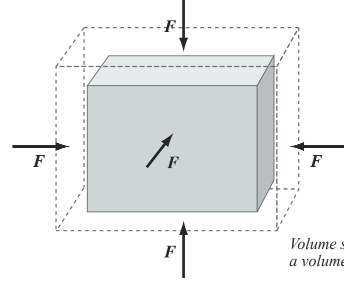
Shear strain produced by a shear stress

$$\text{bulk modulus} = -\frac{F/A}{\Delta V/V}$$

volume stress = $\frac{F}{A}$ volume strain = $\frac{\Delta V}{V}$

F = normal force
A = surface area

ΔV = change in volume
V = original volume



Volume strain produced by a volume stress

S waves are transverse. Solid geological materials can transmit S waves because a solid body can support a shear stress. In liquids, though, a shear stress simply causes the fluid laminae to slide past each other. A liquid supports a volume stress but not a shear stress. For this reason only P waves, which are longitudinal, can transmit through the liquid core.

Heat & Temperature Practice Items

1. 100 ^{o}C converted to Kelvin scale is which of the following?

A. 212 K
B. 273 K
C. 298 K
D. 373 K

2. The amount of heat necessary to raise one gram of a substance one degree of temperature (K or ^{o}C) is called

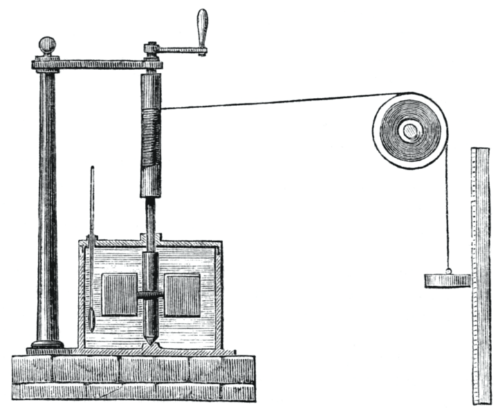
A. one calorie
B. the specific heat capacity
C. the heat capacity of the body
D. the molar heat capacity

3. A floutist tunes her flute in the cool air-conditioned environment of the hotel room 20 °C where the speed of sound through air is approximately 343 m/s. The coefficient linear expansion for steel is 1.1×10^{-5} $(^{o}C)^{-1}$. When she begins to play that evening under the hot stage lighting the temperature on stage will be 25 °C. The speed of sound through air is 346 m/s at 25 °C. When she begins to play

A. Her flute will sound slightly flat.
B. Her flute will sound slightly sharp.
C. Her flute will be in tune.
D. impossible to determine from given information.

4. The figure below shows a contemporary engraving depicting Joule's apparatus for measuring the mechanical equivalent of heat. What temperature change will occur for 100ml of water as the 1kg weight in the apparatus descends 0.8m?

A. 0.02 ^{o}C
B. 0.08 ^{o}C
C. 0.2 ^{o}C
D. 0.4 ^{o}C



5. What is the minimum energy required to transform a 100 g piece of ice at -50 ºC into steam?

C_{ice} = 0.5 cal/g ºC $H_{f(H20)}$ = 79.7 cal/g
$C_{H20(l)}$ = 1.0 cal/g ºC $H_{v(H20)}$ = 539 cal/g
C_{steam} = 0.5 cal/g ºC

A. 15.0 kcal
B. 15.0 kcal
C. 74.5 kca
D. 149 kcal

6. The temperature of the triple point of water

A. decreases with pressure
B. increases with pressure
C. equals 273.16 K
D. is higher than the critical temperature

7. The interior temperature of a house is maintained at 27°C. As compared to the day (17°C) what is the percentage increase in the rate of heat loss at night (7°C) through the walls of the home?

- **A.** 50%
- **B.** 97%
- **C.** 100%
- **D.** 200%

8. Which of the following energy conservation measures for the home minimizes heat losses by conduction?

- **A.** Installation of double-glazed windows
- **B.** Placement of tight-fitting dampers in fireplaces
- **C.** Weather stripping windows and outside doors
- **D.** Use of exhaust fans only when necessary

9. A stone wall is erected across the entire south side of a house behind glass where it can absorb direct solar radiation and serve as a thermal storage mass. It is designed to store 500MJ of thermal energy as it warms up from 16°C at 7 A.M. to 36°F at 5 P.M. The specific heat of the stone used is 0.8 kJ/kg-°C. Find the weight of stone required for the wall.

- **A.** 8.0×10^3 kg
- **B.** 1.4×10^4 kg
- **C.** 3.1×10^4 kg
- **D.** 5.6×10^4 kg

10. If the temperature of the surface of the sun were twice as great, the insolation on the side of the planet Mercury facing the sun would be approximately:

- **A.** 2 times greater
- **B.** 4 times greater
- **C.** 8 times greater
- **D.** 16 times greater

11. An Antarctic observation post is constructed on stilts as a cube with inner dimensions 2m on edge. The four walls, floor and the ceiling are 0.25m thick, constructed from reinforced styrofoam (thermal conductivity: 0.031 W/m·°C). To prevent the temperature inside from dropping below 20°C when the temperature outside is –30°C, what is the approximate minimum generator power required for heating?

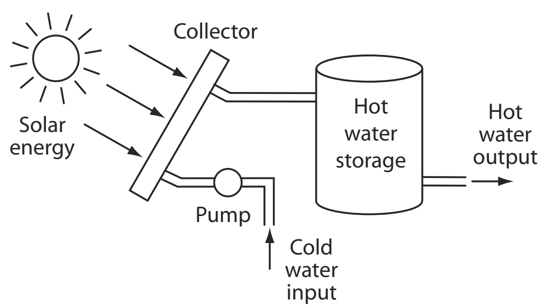
- **A.** 25 W
- **B.** 150 W
- **C.** 600 W
- **D.** 2.4 kW

12. A glass blower's furnace is radiating heat through a 100cm^2 port at a rate of 2400W. What is the approximate temperature inside the furnace? (Stefan's constant = 5.67×10^{-8} W/m^2·K^4)

- **A.** 800 K
- **B.** 973 K
- **C.** 1167 K
- **D.** 1440 K

Passage (Questions 13-17)

Solar water heaters come in a wide variety of designs. Almost all include a collector and storage tank. The performance of a solar collector is affected by numerous factors including absorber plate design, absorber coating, collector glazing, collector insulation, and the orientation of the collector. The ambient air temperature and the intensity of the insolation incident on the collector are additional factors affecting the performance of a solar collector.



Collector efficiency, η, is defined as the ratio of the usable energy output, E_o, to the incident solar radiation, I. There are two primary reasons that a solar collector system will not operate at 100% efficiency. Firstly, not all incident solar radiation is absorbed by the collector. Absorptance, A, of the collector refers to the ratio of absorbed solar energy to incident solar energy. Secondly, because the absorber plate has a temperature greater than the surroundings, absorbed solar energy will leave collector plate by conduction, radiation, and convection. Assuming that heat losses are proportional to the difference between the average temperature of the upper surface of the absorber plate, T_p and the ambient air temperature, T_a, for a particular unglazed collector at a given wind-speed, the thermal loss coefficient, U_L, expressed in W/m²·C°, reflects the combined different modes of heat loss per unit area of the collector as a single critical factor for evaluating flat-plate collector performance.

The expression for the efficiency of an unglazed collector is as follows:

$$\eta = \frac{E_o}{I} = A - \frac{U_L(T_p - T_a)}{I}$$

Unglazed collectors are low-temperature collectors designed to operate at temperatures fairly close to the ambient air temperature. Stagnation temperature refers to the maximum achievable temperature for a solar collector with a stagnant fluid (no motion) at a given ambient wind speed. Efficiency is zero because all of the absorbed energy must be lost to the surroundings. Because their stagnation temperatures are low compared to glazed collectors, unglazed collectors are not usually designed for operating temperatures greater than 5C° to 10C° above ambient temperature.

13. The water flow rate through a bank of solar collectors in a solar water heater is 35 L/hour. The water-in temperature is 30°C, and the water-off temperature is 45°C. What is the power output of this solar water heater?

A. 126 W
B. 146 W
C. 525 W
D. 610 W

14. A 5m length of copper pipe runs from a solar collector panel to a hot water storage tank. At 8 A.M. the pipe temperature is 10°C. At 5 P.M. the temperature has increased to 50°C. Given that the coefficient of linear expansion for copper is 1.7×10^{-5}, how much will the length of the pipe have increased?

A. 0.4 mm
B. 3.4 mm
C. 6.8 mm
D. 3.4 cm

15. A solar collector system is 100% efficient if

A. energy losses from the collector are zero
B. the absorptance of the collector is 100%
C. usable energy output equals incident solar energy
D. the collector temperature equals ambient temperature

16. Which of the following would increase subsequent to the installation of a glass plate and casement over the absorber plate of a solar collector system?

A. the amount of solar radiation reaching the collector plate
B. the stagnation temperature of the solar collector system
C. the thermal loss coefficient of the collector system
D. convection losses from the collector plate

17. At the stagnation temperature of a solar collector system

A. water stops flowing through the collector.
B. the maximum temperature for the collector is reached for given insolation and ambient wind conditions.
C. the temperature is 5C° to 10C° above ambient temperature for glazed collectors.
D. usable energy equals absorbed incident solar energy.

Heat & Temperature
Answers and Explanations

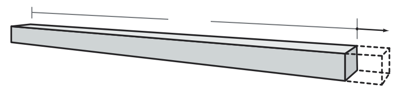
1. D
The Kelvin temperature represents the true thermodynamic temperature. The Kelvin temperature is directly proportional to the kinetic energy of the particles of a particular substance (the constant of proportionality depends on the molar heat capacity of the substance). Zero on the Kelvin scale is called absolute zero because it is the lowest possible temperature. Molecular motion ceases. The size of a Kelvin unit was developed to agree with the centigrade scale (100 gradations between the freezing and boiling points of water), so in terms of the unit magnitude: 1 K = 1°C. However, the Kelvin scale is offset 273.15.

$$T = T_c + 273.15$$

2. B
The specific heat is the heat capacity per unit mass. Specific heat tells us how many joules (or calories) are required to raise one gram of a substance one degree Celsius. The unit of heat, the calorie, is defined in terms of the specific heat of liquid water. The specific heat of water is 1 cal/g°C.

3. B
There are two things happening which will affect the pitch of the flute. Thermal expansion alters the length of the flute and the greater temperature increases the speed of sound in air.

Thermal expansion occurs when the spacial dimensions of a solid increase when it is heated. For small temperature changes, the change in length is directly proportional to the original length and the temperature change. The constant of proportionality for a particular material, the coefficient of linear expansion, α, is multiplied by the original length and temperature change to determine the change in length.



$$\Delta l = \alpha\, l_0\, \Delta T$$

Because the coefficient of linear expansion for steel given in the problem is $1.1 \times 10^{-5}\ (^\circ\mathrm{C})^{-1}$. A 5° increase in temperature will produce a bit more than half of 1/100 of a 1% increase in the length of the flute. Lengthening the flute lengthens the wavelength of the fundamental mode. (A flute is a pipe open at one end. $\lambda_{fund} = 4L$).

By itself the slightly longer wavelength would cause the flute to found flatter. However, despite the slightly longer wavelength the frequency is still going to be higher than before. This is because the approximately 1% increase in wavespeed in the warmer environment will increase the frequency much more than the flattening effect of the lengthening flute.

$$f = \frac{v}{\lambda_n} = \frac{n}{4L}v \qquad (n = 1,3,5,\ldots)$$

4. A
Joule's experiment is one of the classics from the history of physics demonstrating conservation of energy. As the weight descends its gravitational potential energy is transformed into the work the paddles do on the water in which they are immersed. The work of the paddles is dissipated through friction into thermal energy. Joule demonstrated that the mechanical work of the paddles is quantitatively equivalent to heat flow.

First, we need to determine the initial potential energy of the weight.

$$U = mgh$$

$$U = (1\ \mathrm{kg})(10\ \mathrm{m/s^2})(0.8\ \mathrm{m})$$

$$U = 8\ \mathrm{J}$$

Now we determine the change in temperature brought about by the addition of 8 J thermal energy in 100 g of water.

$$Q = mc\Delta T$$

$$\Delta T = \frac{Q}{mc}$$

$$\Delta T = \frac{8 \text{ J}}{(100 \text{ g})(4.18 \text{ J g}^{-1}\,{}^{\circ}\text{C}^{-1})}$$

$$\Delta T = 0.02\,{}^{\circ}\text{C}$$

5. C

The path has four steps: 1) heating the ice from -50°C to 0°C 2) melting the ice 3) heating the liquid water from 0°C to 100°C; and finally 4) boiling the water. The amount of heat which must be added for temperature change equals the product of the mass, specific heat and temperature change. For the phase changes, the amount of heat equals the product of the mass and the heat of transformation. Here are the computations for each of the four steps:

$$Q_{-50^{\circ}\text{C}\rightarrow 0^{\circ}\text{C}} = (100 \text{ g})\left(.50 \frac{\text{cal}}{\text{g}\,{}^{\circ}\text{C}}\right)(50\,{}^{\circ}\text{C}) = 2500 \text{ cal}$$

$$Q_{\text{fusion}} = (100 \text{ g})\left(80 \frac{\text{cal}}{\text{g}}\right) = 8000 \text{ cal}$$

$$Q_{0^{\circ}\text{C}\rightarrow 100^{\circ}\text{C}} = (100 \text{ g})\left(1 \frac{\text{cal}}{\text{g}\,{}^{\circ}\text{C}}\right)(100\,{}^{\circ}\text{C}) = 10{,}000 \text{ cal}$$

$$Q_{\text{vaporization}} = (100 \text{ g})\left(540 \frac{\text{cal}}{\text{g}}\right) = 54{,}000 \text{ cal}$$

$$\text{sum} = 74{,}500 \text{ cal}$$

6. C

The triple point of a substance is the temperature and pressure at which the three phases of that substance coexist in thermodynamic equilibrium. It is a specific temperature and a specific pressure. The triple point of water is at 0.01°C (273.16K) and 611.2Pa (4.58 torr).

7. D

The rate of heat flow through the walls by conduction is proportional to the difference between the temperatures inside and outside the house.

$$\frac{Q}{t} = K\,A\,\frac{\Delta T}{\Delta x}$$

- Q = heat flow
- t = time
- K = thermal conductivity of material
- A = cross-sectional area
- ΔT = temperature difference across the conductor $(T_2 - T_1)$
- Δx = conductor thickness

During the day the temperature difference, ΔT, is 10°C. At night it is 20°C. Doubling the temperature difference at night results in a 100% increase in the rate of heat flow by conduction.

8. A

The space between the panes of glass in double glazed windows is very effective at reducing heat flow by conduction. Occupying the space is a gas with a very low thermal conductivity. The lower the thermal conductivity, K, the lower the rate of heat flow by conduction.

$$\frac{Q}{t} = K\,A\,\frac{\Delta T}{\Delta x}$$

- Q = heat flow
- t = time
- K = thermal conductivity of material
- A = cross-sectional area
- ΔT = temperature difference across the conductor $(T_2 - T_1)$
- Δx = conductor thickness

The other choices represent methods to decrease the rate of heat loss through infiltration, in other words, the bulk movement of air from the home.

9. C

The thermal energy stored by the wall is proportional to the mass of the wall, the specific heat of the stone, and the temperature change.

$$Q = mc\Delta T$$

$$m = \frac{Q}{c\Delta T}$$

$$m = \frac{5 \times 10^{5} \text{ kJ}}{(0.8 \text{ kJ kg}^{-1}\,{}^{\circ}\text{C}^{-1})(20\,{}^{\circ}\text{C})}$$

$$m = 31{,}250 \text{ kg}$$

10. D

Radiation is heat flow by electromagnetic waves. The rate of heat flow, $\frac{Q}{t}$, from a body by radiation is proportional to the *fourth power* of the temperature. (The rate also depends on the surface area of the body and the emissivity, ε, of the material. σ is Stefan-Boltzmann constant (5.7×10^{-8} W/m^2 · K^2))

$$\frac{Q}{t} = A\varepsilon\sigma T^4$$

Doubling temperature leads to a *sixteen fold* increase in the rate of emission of radiation.

$$\left(\frac{Q}{t}\right)_{\text{new}} = A\varepsilon\sigma\,(2T)^4 = 16A\varepsilon\sigma T^4$$

11. B

The heating power required will equal the rate of heat loss through the walls by conduction:

$$\frac{Q}{t} = K\,A\,\frac{\Delta T}{\Delta x}$$

$$\frac{Q}{t} = (3.1 \times 10^{-2}\ \text{W/m·°C})(2.4 \times 10^{1}\ \text{m}^2)\ \frac{5 \times 10^{1}\ \text{°C}}{2.5 \times 10^{-1}\ \text{m}}$$

$$\frac{Q}{t} = 149\ \text{W} \sim 150\ \text{W}$$

12. D

First, let's convert the size of the furnace port into SI units, from cm^2 to m^2.

$$100\ \text{cm}^2 \bullet \left(\frac{1 \times 10^{-2}\ \text{m}}{\text{cm}}\right)\left(\frac{1 \times 10^{-2}\ \text{m}}{\text{cm}}\right)$$

$$= 1 \times 10^{-2}\ \text{m}^2$$

The port of the furnace is a reasonable approximation of a blackbody perfect emitter (ε = 1). The rate of heat flow by radiation is proportional to the surface area of the body, the emissivity, ε, of the material and the fourth power of the temperature.

$$\frac{Q}{t} = A\varepsilon\sigma T^4$$

Q = heat (light) emitted
t = time
A = surface area of emitter
ε = emissivity
σ = Stefan-Boltzmann constant (5.7×10^{-8} W/m^2 · K^2)
T = emitter absolute temperature

Solving for the temperature of our glass blower's furnace is a good exercise in operations of scientific notation and mental math.

$$\frac{Q}{t} = A\varepsilon\sigma T^4$$

$$2.4 \times 10^3\ \text{J/s} = (10^{-2}\ \text{m}^2)(1)(6 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^2)\ T^4$$

$$T^4 = \frac{2.4 \times 10^3}{6 \times 10^{-10}}$$

$$T^4 = 4 \times 10^{12}$$

$$T = 1.4 \times 10^3\ \text{K}$$

Taking the 4th root of 4×10^{12} is the same as taking the square root of the square root. Also, simplifying the Stefan-Boltzmann constant to make our math easier landed us a little lower than the answer choice. When you do mental math and simplify numbers, keep track of the direction the error is going to propagate. This will help you feel comfortable with your answer choice and may help you distinguish the correct answer if you land between two choices. This usually won't be a problem, though, because numerical answer choices on the MCAT are almost always spaced far apart. The MCAT encourages mental math.

13. D

Because the specific heat of water is close to mind expressed on a per gram basis, let's convert our flow rate from L/hr to ml/s. Remember 1 ml$_{H_2O}$ = 1 g$_{H_2O}$, so this will also be g/s.

$$\left(\frac{35\ \text{L}}{\text{hr}}\right)\left(\frac{\text{hr}}{3600\ \text{s}}\right)\left(\frac{1000\ \text{ml}}{\text{L}}\right) \approx \frac{10\ \text{ml}}{\text{s}} = \frac{10\ \text{g}}{\text{s}}$$

10g of water is flowing through the system per second, leaving 15°C warmer. Because the specific heat of water is 1 cal g^{-1} $°C^{-1}$, it's very good if you see 10g of water increasing 15°C and say, 'that's 150 calories', and because 1 cal = 4.18 J, you know then that's about 600 J, so the power output of the solar water heater is 600 J/s or about 600 W.

$$\left(\frac{10\text{ g}}{\text{s}}\right)\left(\frac{4.18\text{ J}}{\text{g °C}}\right)\left(15\text{ °C}\right) \approx 600\text{ W}$$

14. B

The coefficient of linear expansion, α, is multiplied by the original length and temperature change to determine the change in length.

$$\Delta l = \alpha\, l_0\, \Delta T$$

$$\Delta l = (1.7 \times 10^{-5})\,(5\text{ m})\,(40\text{ °C})$$

$$= 3.4 \times 10^{-3}\text{ m}$$

15. C

In the passage collector efficiency is defined as the ratio of the usable energy output to the incident solar radiation. 100% efficiency would only occur if *both* choices 'B' and 'A' are true, ie. the absorptance of the collector is 100% and energy losses from the collector are zero.

16. B

The stagnation temperature is the temperature at which all absorbed energy is lost to the surroundings. The glass plate and casement would decrease the rate of conduction losses from the collector plate to the environment for a given temperature difference much as a double-glazed window decreases conduction losses from a heated building to the outside air. Therefore, our water heater system would require a higher collector temperature for the rate of energy loss to equal the rate of absorption if the collector were covered with a glass plate and casement.

17. B

The rate of heat loss from the system by conduction to the surroundings increases with temperature. At the stagnation temperature, the rate of heat loss has grown to equal the rate of absorption. It is the maximum temperature for the collector for given insolation and ambient wind conditions.

Ideal Gas & Kinetic Theory Practice Items

1. Which of the following volumes is closest to that occupied by one mole of an ideal gas at standard state temperature and standard pressure?

A. 1 liter
B. 18.9 liters
C. 22.4 liters
D. 24.4 liters

2. A constant volume gas thermometer will be most accurate

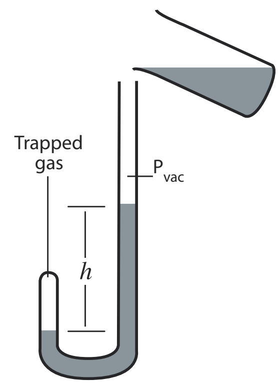
A. at low pressures
B. at low temperatures
C. when conditions for the gas are near the condensation stage
D. if the gas is high density

3. In the interstellar regions of the Milky Way, a temperature of 2.7 K arises from radiation and particle kinetic energy. Matter exists in the form of approximately one million neutral hydrogen atoms per cubic meter. Which is the best estimate of the pressure in interstellar space?

A. 3.5×10^{-19} Pa
B. 3.6×10^{-17} Pa
C. 2.2×10^{-7} Pa
D. 2.7×10^{-6} Pa

4. The U-tube pictured below contains a volume, V_g, of ideal gas trapped by a column of mercury under a vacuum. As mercury is added to the U-tube with the system maintained at constant temperature,

A. the product $V_g h$ maintains a constant value as h increases.
B. the product $V_g h$ decreases.
B. the product $V_g h$ increases.
C. heat flow occurs into the gas.



5. Which of these gases has the lowest molar heat capacity?

A. Ar
B. H_2
C. NH_3
D. C_3H_8

6. What is the approximate ratio of the specific heat (cal/g, constant volume) of neon gas (MW 20.2 g) to the specific heat of krypton gas (MW 83.8 g)?

A. 1:4
B. 1:1
C. 4:1
D. not enough information to determine

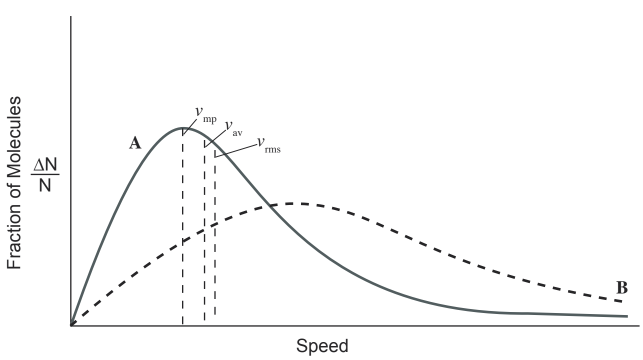
7. Almost all atoms in a sample of helium gas become completely ionized when the thermal energy reaches approximately 3.5×10^3 kJ/mol. At what approximate temperature does helium gas completely ionize?

A. 3,500 K
B. 5,250 K
C. 10,500 K
D. 28,000 K

Passage (Questions 8-12)

Gas molecules move rapidly in all directions in a random fashion. For a given temperature, the Maxwell-Boltzmann distribution describes the variation of speeds among the molecules. The distribution is often represented graphically as a plot of the fraction of molecules vs. speeds. The fraction of molecules with very high speeds or very low speeds is small. The peak of the graph corresponds to the most probable speed. A somewhat higher speed than the most probable speed is the root mean square speed, RMS, which corresponds to the speed of a particle with average kinetic energy. The average speed itself is $0.913 \times$ RMS.

The graph below illustrates the Maxwell-Boltzmann distributions for two gases of equal molecular size present at equimolar concentrations in two separate stoppered flasks. The distribution for gas A is shown with the solid line. The distribution for gas B is shown with the dotted line.



8. In comparison to gas A, Gas B

A. is at a higher temperature.
B. is at a lower temperature.
C. has larger molecules.
D. has smaller molecules.

9. In a sample of gas more molecules have this particular speed than any other.

A. v_{mp}
B. v_{av}
C. v_{rms}
C. impossible to determine

10. Assuming the temperature of gas A were 300K, what is the approximate temperature of gas B?

A. 150 K
B. 425 K
C. 600 K
D. 1200K

11. A graph illustrating the Maxwell-Boltzmann distributions of which of the following two gases at equimolar concentrations in thermal equilibrium would most closely resemble the graph in the passage?

A. O_2 and N_2
B. Ar and Ne
C. CH_4 and He
D. NH_3 and H_2

12. In kinetic theory the mean free path of a particle is the average distance the particle travels between collisions with other moving particles. For gas particles it may be shown that the mean free path, in meters, is

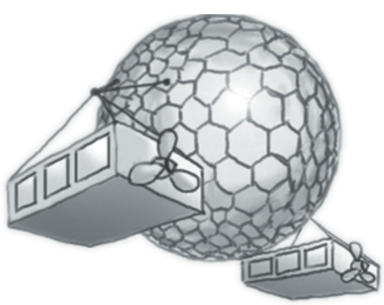
$$l = \frac{k_b T}{\sqrt{2}\pi d^2 P}$$

where k_b is the Boltzmann constant in J/K, T is the temperature in K, P is pressure in Pascals, and d is the diameter of the gas particles in meters.

Which of the following causes an increase in the mean free path of the particles of a gas?

A. increasing the temperature of the gas at constant volume
B. increasing the pressure of the gas at constant volume
C. increasing the volume of the gas at constant temperature
D. ionizing the gas with an electron beam

Passage (Questions 13-16)



A large vacuum dirigible, such as depicted in the artist's rendering above, would produce enormous lift. A spherical vacuum dirigible constructed of a geodesic sphere 200m in diameter with 1.2×10^6 kg devoted to its aluminum frame and carbon fiber skin with 0.5atm internal pressure would be capable of floating in the Earth's atmosphere carrying approximately 1.5×10^6 kg in equipment, cargo and passengers in addition to the mass of its structure and cladding.

Current techniques employed in the design and construction of geodesic domes could produce an extremely stable geodesic sphere as described, prior to evacuation. However, the true challenge would be to construct an airship that could withstand the compressive forces after the vacuum has been introduced. The difficulty of the engineering challenges involved can be appreciated in the determination of the material constraints for a vacuum dirigible comprised of a homogeneous spherical shell enclosing a total vacuum. The total force on a spherical shell of radius R by an external pressure P is $\pi R^2 P$. The force on each hemisphere in equilibrium along the equator will produce the compressive stress given below:

$$\sigma = \frac{\pi R^2 P}{2\pi R h} = \frac{RP}{2h}$$

where h is the shell thickness.

Neutral buoyancy occurs when the shell has the same mass as the displaced air, which occurs when $h/R = \rho_a/(3\rho_s)$, where ρ_a is the air density and ρ_s is the shell density. Combining with the stress equation gives

$$\sigma = \frac{3\rho_s P}{2\rho_a}$$

For terrestrial conditions such a degree of stress is of the same order of magnitude as the compressive strength of aluminum alloys, arguing for the feasibility of the spherical shell design.

Unfortunately this disregards buckling. Using the formula for the critical buckling pressure of a sphere

$$P_{cr} = \frac{2Eh^2}{\sqrt{3(1-\mu^2)}} \frac{1}{R^2}$$

where E is the modulus of elasticity and μ is the Poisson ratio of the shell material, ie. the relationship of transverse bulging to axial compression. (Most potential shell materials possess a Poisson ratio of approximately 0.3). Substituting the condition for neutral buoyancy, $h/R = \rho_a/(3\rho_s)$, gives a necessary condition for a feasible shell:

$$\frac{E}{\rho_s^2} = \frac{9P_{cr}\sqrt{3(1-\mu^2)}}{2\rho_a}$$

The requirement is about 4.5×10^5 kg^{-1}m^{5}s^{-2}. This cannot even be achieved using diamond ($E/\rho_s^2 \approx 1.0 \times 10^5$ kg^{-1}m^{5}s^{-2}).

In summary, a vacuum dirigible comprised of a homogeneous spherical shell does not appear to be possible given currently available materials. However, dropping the assumption that the shell is a homogeneous material may allow lighter and stiffer structures such as with a honeycomb structure or geodesic construction. A number of engineering groups have claimed success in creating viable designs in recent years although there have been no public demonstrations of a working prototype.

13. A vacuum dirigible could lose buoyancy through the influx of gas molecules through pinhole defects in its carbon fiber skin. Assuming the airship material and air density remained unchanged as temperature increased, the increase in the rate of gas influx in moving from a 0°C environment to a 30°C environment would be approximately

A. 3%
B. 10%
C. 17%
D. 81%

14. A hypothetical vacuum airship achieves neutral buoyancy with an interior pressure of 0.5 atm. What approximate interior temperature within 27°C surroundings would a hot-air balloon of the same volume and mass need to establish to generate the same lift?

A. 54°C
B. 129°C
C. 254°C
D. 327°C

15. According to the passage, construction of a viable vacuum airship comprised of a homogeneous spherical shell might be constructed if a material with the following properties compared to diamond could be developed:

A. 1/2 the rigidity and 1/4 the density
B. the same rigidity and 1/2 the density
C. twice the rigidity with the same density
D. twice the rigidity and twice the density

16. A vacuum dirigible constructed of a homogeneous spherical shell is to be developed for flight within a dense low pressure extra-terrestrial atmosphere. From the information presented in the passage it can be deduced that increasing the design radius

A. would require an increase in the thickness of the shell material directly proportional to the increased radius.
B. would lead to more favorable material constraints as radius increased.
C. would enable the dirigible to operate in higher temperatures.
D. would make it more possible for the dirigible to be constructed from a material having a low Poisson ratio.

Ideal Gas & Kinetic Theory
Answers and Explanations

1. D

One mole of an ideal gas occupies 22.4 L at STP. This fact you absolutely must memorize for the MCAT. The temperature at STP is 273 K. This is standard temperature. However, 273 K is not *standard state temperature*. Standard state temperature is not 273 K but 298K. This may seem ridiculous, but there is a figure of merit to knowing about this difference. Many bench-top measurements are at 298 K not 273 K. Charles' Law tells us that at constant pressure the volume of an ideal gas sample is directly proportional to its temperature, i.e. V/T = constant, so at 298 K the volume will be greater than 22.4 L.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{22.4\text{ L}}{273\text{ K}} = \frac{24.4\text{ L}}{298\text{ K}}$$

2. A

This question is really asking 'when will a real gas behave most like an ideal gas?' Generally, a gas behaves more like an ideal gas at higher temperature and lower pressure, as the potential energy due to intermolecular forces becomes less significant compared with the particles' kinetic energy, and the size of the molecules becomes less significant compared to the empty space between them.

3. B

To use the ideal gas law, first we need to know how many moles is represented by one million atoms.

$$1 \times 10^6 \text{ particles} \left(\frac{1 \text{ mole}}{6.02 \times 10^{23} \text{ particles}} \right)$$

$$= 0.16 \times 10^{-17} = 1.6 \times 10^{-18} \text{ mole}$$

Now we can determine the pressure. Because the answer choices are spaced numerically, you have plenty of latitude for mental math with these computations.

$$PV = nRT$$

$$P = \frac{nRT}{V}$$

$$P = \frac{(1.6 \times 10^{-18} \text{ mole})(8.3 \text{ J mole}^{-1} \text{ K}^{-1})(2.7 \text{ K})}{1 \text{ m}}$$

$$P = \frac{(1.6 \times 10^{-18} \text{ mole})(8.3 \text{ J mole}^{-1} \text{ K}^{-1})(2.7 \text{ K})}{1 \text{ m}}$$

$$P = 3.6 \times 10^{-17} \text{ Pa}$$

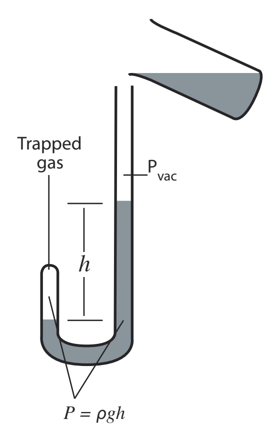
4. A

Boyle's Law:

$$P_1V_1 = P_2V_2 \quad \text{constant temperature}$$

At constant temperature the pressure of an ideal gas sample is inversely proportional to its volume, i.e. PV = constant.

The pressure in our trapped gas is directly proportional to the height of the mercury column.



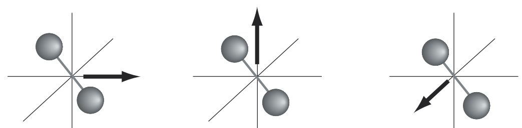
$$P_gV_g = \text{constant} \qquad P_g = \rho gh$$

$$\rho g h V_g = \text{constant}$$

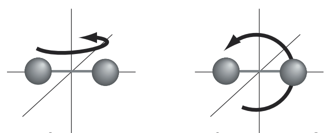
$$hV_g = \text{constant}$$

5. A

A monatomic gas molecule such as argon possesses only kinetic energy deriving from its linear motion. A diatomic gas molecule, like H_2, in addition to translational motion, can also rotate and vibrate.



translational motion - three degrees of freedom



rotational motion - two degrees of freedom

vibration - two degrees of freedom

Storing energy in both vibrational and rotational modes, a diatomic gas has more partitions for thermal energy. As a sample of diatomic gas takes in heat, the energy spreads out into all of its degrees of freedom. The other choices besides argon can absorb heat flow into all modes, translational, rotational, and vibrational. For this reason, the molar heat capacity of the other gaseous substances is greater than the molar heat capacity of argon.

6. C

Because neon and krypton are both monatomic gases. They both possess kinetic energy at the particle level, ie. thermal energy, only in the form of translational kinetic energy. There are no rotational or vibrational modes. If two substances have the same number of places to put kinetic energy at the particle level, they will have nearly the same molar heat capacity. In the case of neon and krypton, their respective constant volume molar heat capacities are very close to the ideal gas value.

$$C_V = \frac{3}{2}R = 12.5\ \frac{\text{J}}{\text{mol K}}$$

If their molar heat capacities are very close (joules per mole degree Kelvin), then their specific heats will be different (joules per gram degree Kelvin) according to molecular weight.

$$c_{\text{neon}} = \left(\frac{12.5\ \text{J}}{\text{mol K}}\right)\left(\frac{\text{mol}}{20.2\ \text{g}}\right)$$

$$c_{\text{kryp}} = \left(\frac{12.5\ \text{J}}{\text{mol K}}\right)\left(\frac{\text{mol}}{83.8\ \text{g}}\right)$$

$$c_{\text{neon}} : c_{\text{kryp}} \approx 4:1$$

7. D

Apply the ideal gas model. Helium, a monatomic noble gas, is close to ideal gas behavior. In a sample of an ideal gas, internal energy is only in the form of thermal energy. The internal energy of an ideal gas is directly proportional the temperature.

$$U = \tfrac{3}{2}nRT$$

$$3.5 \times 10^6\ \text{J} = \tfrac{3}{2}\ (1\ \text{mol})\ (8.3\ \frac{\text{J}}{\text{mol K}})\ T$$

$$T = 2.8 \times 10^5\ \text{K}$$

8. A

Gas B is at a higher temperature. As the temperature of the molecules represented by a Maxwell-Boltzmann distribution increases, the distribution flattens out. rms speed is higher. This corresponds to a greater average kinetic energy per particle, a higher temperature.

9. A

The speed at the top of the curve is called the most probable speed because the largest number of molecules have that speed.

10. D

The temperature of ideal gas is directly proportional to the translational kinetic energy of the particles. Real gases may have thermal energy in the form of rotational and vibrational kinetic energies as well, so with real gases it's more proper to say that the relationship is between temperature and kinetic energy per degree of freedom within translational, rotational, and vibrational modes. Nevertheless, whether the gas is ideal or real, temperature increases with the average translational kinetic energy per particle.

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

The particles in gas B are moving at approximately twice the rms speed, so the average kinetic energy per particle is four times greater and so is the temperature.

11. C

For this question we assume the distribution represents two gases in thermal equilibrium. In other words, they are the same temperature. At the same temperature, the average translational kinetic energy per particle will be the same, yet the particles of gas B are moving twice as fast on average.

$$\frac{1}{2}m_A\overline{v_A^2} = \frac{1}{2}m_B\overline{v_B^2}$$

If gas B particles are moving twice as fast, to possess the same translational kinetic energy, the mass of gas B particles must be ¼ the mass of gas A particles. The only pair satisfying this condition are CH_4 (MW 16u) and He (MW 4u).

12. C

Neither ‘A’ nor ‘B’ is correct because temperature and pressure go up or down together in direct proportionality at constant volume. Both choices ‘A’ and ‘B’ would produce no change in the mean free path. Choice ‘C’ however would increase the mean free path because an increase in volume at constant temperature would lead to a decrease in pressure.

13. A

Comparing gases at the same temperature, Graham found experimentally that the rate of effusion of a gas is inversely proportional to the square root of the masses of the particles. This is because effusion rate is proportional to the rms speed of the particles. In this problem, the temperature of the gas increased. A difference of 0°C and 30°C is a difference of 273K and 303K. The Kelvin temperature increased approximately 10%. The average translational kinetic energy of the gas particles is directly proportional to the Kelvin temperature.

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

A 10% increase in Kelvin temperature corresponds to a 10% increase in translational kinetic energy, which, in turn, corresponds to approximately a 3% increase in rms speed ($\sqrt{10}$).

14. D

For the buoyant force to be equal and opposite to the weight of the aircraft in both cases (neutral buoyancy), the air inside the hot-air balloon would need to weigh the same as the air inside the vacuum dirigible at 0.5 atm. In other words, it needs to be half as dense as the surrounding air. In a hot air balloon, the air within the balloon is the same pressure as the surrounding air. To possess the same pressure and half the density, the Kelvin temperature needs to be twice as great. You can see this in the ideal gas law.

$$PV = nRT$$

$$\frac{n}{V} = \frac{P}{TR}$$

We are given a temperature for the surroundings of 27°C, which equals 300K ($T = T_c + 273.15$). Doubling 300K to 600K and converting back to Celsius gives us 327°C.

15. A

This passage has a number of difficult out-of-scope elements. It’s important to remember in the exam that a passage like this isn’t about foreknowledge of the out-of-scope elements. Ultimately, the questions are going to be about fundamentals and how well you kept your footing. An important figure of merit when you see an unfamiliar equation is to get yourself on speaking terms with it. What is the equation saying? What changes with what?

An elastic modulus indicates how difficult a material is to deform under stress. A material possessing a high elastic modulus has a high rigidity. In the passage we are told that the ratio of elastic modulus to the square of density of diamond is $E/\rho_s^2 \approx 1.0 \times 10^5$ kg^{-1}m^{5}s^{-2}, but the minimum requirement to prevent buckling for a vacuum dirigible of neutral buoyancy is presented as $E/\rho_s^2 = 4.5 \times 10^5$ kg^{-1}m^{5}s^{-2}. If a material were developed where the elastic modulus were halved while also quartering density that would increase E/ρ_s^2 eight-fold, so while this material would be less rigid than diamond its lower density (and thus weight) would reduce the stresses it would need to undergo and thus make a viable vacuum dirigible possible.

16. A

Neutral buoyancy determines a ratio of shell thickness to radius as a function of the ratio of the density of the surrounding air to the density of the shell material.

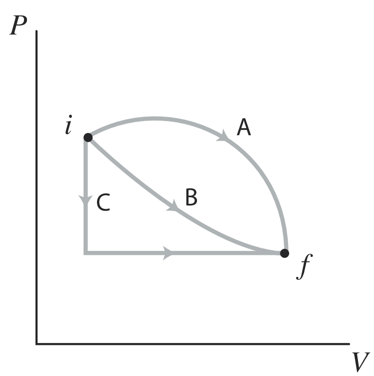
$$h/R = \rho_a/(3\rho_s)$$

As the radius increases the shell thickness must increase in direct proportion.

First Law of Thermodynamics Practice Items

1. The graph below show three alternative paths involving changes in the pressure and volume of an ideal gas to transform it from an initial state, i, to a final state, f. Rank the paths in order of greatest change in internal energy.

A. A > B > C
B. B > C > A
C. C > B > A
D. All three are equal.



2. The temperature is somewhat higher in the final state than the initial state in the graph above. Rank the paths in terms of the magnitude of heat flow.

A. A > B > C
B. B > C > A
C. C > B > A
D. All three are equal.

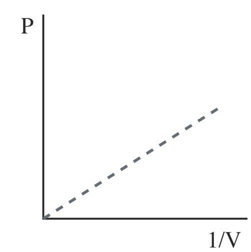
3. Which of the following must **not** be true for an isothermal compression of an ideal gas

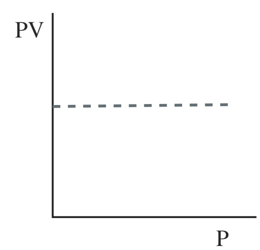
A. The work done on the gas equals the heat flow in magnitude.
B. Internal energy increases.
C. Particles have the same average kinetic energy before and after the expansion.
D. The pressure-volume product is constant.

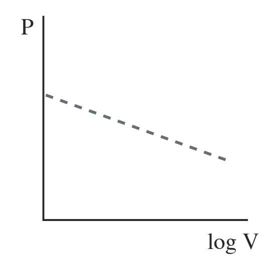
4. An ideal gas within a well insulated container expands and does work against a piston. Which of the following provides the microscopic basis for why the gas loses internal energy?

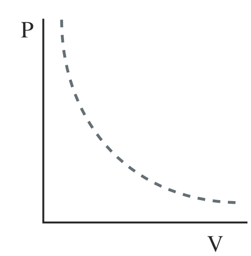
A. Accelerating charges emit electromagnetic radiation.
B. The particle collisions with the piston are elastic.
C. A gas particle loses speed in colliding with a receding wall.
D. Temperature is proportional to the average translational kinetic energy of the particles.

5. Which of the graphs below does **not** represent an isotherm for an ideal gas?

A.


B.


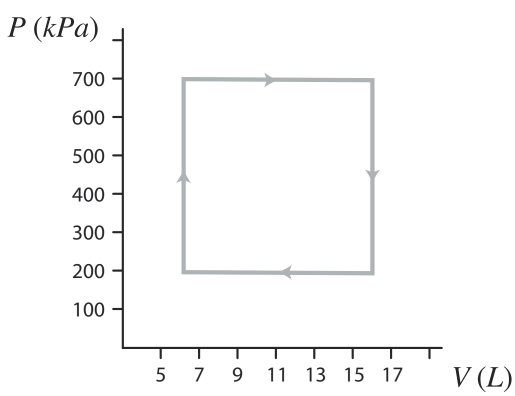
C.


D.


6. The heat of vaporization of water (cal/g)

A. equals the internal energy change per gram of water in transforming from liquid to gas at 1 atm of pressure.
B. is greater than the internal energy change per gram of water in transforming from liquid to gas at 1 atm of pressure.
C. equals the electrostatic potential energy increase among water molecules along lines of intermolecular force.
D. results in an increase in the kinetic energy of water vapor molecules at 100ºC compared to molecules of liquid water at 100ºC.

7. The powerful engine takes in heat both isochorically and during an isobaric expansion. Then the engine expels heat isochorically and subsequently during an isobaric compression as it returns to the initial state. The engine cycle is represented in the diagram below:



What is the difference between the heat taken en in by this engine and the heat that the engine expels each cycle?

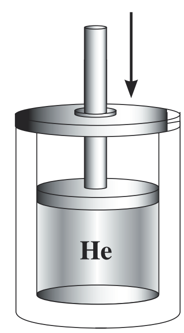
A. 0 J
B. 50 J
C. 3500 J
D. 5000 J

8. In biochemical processes enthalpy change and internal energy change are often essentially equal because

A. most biochemical processes are isovolumetric.
B. biochemical processes are most likely to be carried out at constant pressure.
C. biochemical processes are often coupled.
D. biochemical processes are most likely to be carried out at constant temperature.

9. 1.1 L of helium at STP (C_v = 12.5 J mol^{-1} K^{-1})is compressed adiabatically until the temperature of the gas reaches 313 K. How much work was performed in the compression?

A. 25 J
B. 196 J
C. 250 J
D. 400 J



10. 2.0 mol of H_2 and 1.0 mol of O_2 react to completion at 200°C and 1 atm, producing 2.0 mol of gaseous water at 200°C and 1 atm. A total of 485kJ is evolved. How much heat evolves with the same reaction taking place at 200°C within a bomb calorimeter of fixed volume?

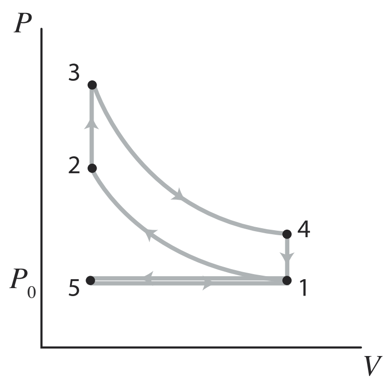
A. 483 kJ
B. 485 kJ
C. 487 kJ
D. 507 kJ

Passage (Questions 11-16)

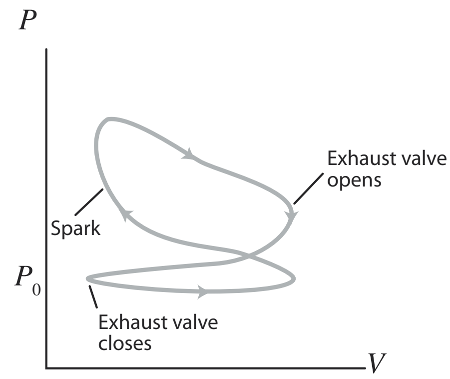
Within an internal combustion engine the combustion of a fuel occurs with an oxidizer (usually air) in a combustion chamber that is an integral part of the working fluid flow circuit. An Otto cycle is an idealized thermodynamic cycle which describes the functioning of a typical spark ignition reciprocating piston engine, the type of internal combustion engines most commonly employed in automobiles.

To conduct elementary thermodynamic analyses of internal combustion engines, considerable simplification is required. To simplify the analysis, standard assumptions are made. Gas and air mixture are modeled as an ideal gas air-standard. All the processes making up the cycle are internally reversible. The combustion process is replaced by a heat-addition process from an external source.

On the diagram of the Otto cycle shown below, the cycle begins in state 5 with an isobaric expansion. This is followed by an adiabatic compression stroke. Heat is added in an isochoric process with the combustion of fuel, followed by an adiabatic expansion process, which characterizes the power stroke. Isochoric cooling is then followed by the exhaust stroke.



A detailed study of the performance of an actual gas power cycle is rather complex and accurate modeling of internal combustion engines normally involves computer simulation. The following diagram represents an actual internal combustion cycle. Notice that an actual cycle does not have the sharp transitions between the different processes of the ideal cycle.



Under the cold air-standard assumption, the thermal efficiency, η_{th}, of the ideal Otto cycle is

$$\eta_{th} = 1 - \frac{C_V (T_4 - T_1)}{C_V (T_3 - T_2)}$$

where C_V is the heat capacity at constant volume, and T_1, T_2 etc. are the temperatures corresponding to the various states 1,2,3 & 4 in the cycle represented in the ideal Otto cycle diagram.

We can simplify the above expression using the fact that the processes from 1 to 2 and from 3 to 4 are adiabatic.

$$\eta_{th} = 1 - \frac{1}{(V_1/V_2)^{\lambda-1}} = 1 - \frac{1}{r^{\lambda-1}}$$

λ is the ratio of the constant pressure heat capacity, C_p, to the heat capacity at constant volume C_V. ($\lambda_{air} = 1.4$). The term $V_1/V_2 = r$ is called the compression ratio. It is evident that the Otto cycle efficiency depends directly upon the compression ratio.

11. The exhaust and intake strokes in the Otto cycle

A. involve adiabatic compression followed by adiabatic expansion.
B. take place at constant temperature.
C. involve heat flow in an isochoric process.
D. require no net thermodynamic work.

12. An internal combustion engine with a compression ratio of 8.0 utilizes an air fuel mixture with a heat capacity ratio, C_P/C_V of 1.33. What percentage of the thermal energy produced in the combustion reaction can be transformed into useful work?

A. 16%
B. 50%
C. 75%
D. 84%

13. During the compression stroke in the Otto cycle

A. no heat flow occurs and temperature increases in the piston
B. heat flow occurs from the piston and temperature decreases
C. temperature remains constant
D. heat flow occurs into the piston and temperature increases

14. The heat of combustion is defined in chemistry as the energy released as heat flow when a compound undergoes complete combustion with oxygen under standard conditions. Assuming the complete combustion of the fuel-air mixture in an internal combustion engine, the thermal energy introduced into the piston by combustion during isochoric pressurization in the Otto cycle

A. will be greater per mole of fuel than the heat of combustion of the fuel
B. is less per mole of fuel than the heat of combustion of the fuel
C. equals the heat of combustion of the fuel
D. may be less or greater than the heat of combustion depending on the compression ratio of the engine

15. According to the assumptions underlying the Otto cycle described in the passage above, which of the following expressions represents the thermal energy liberated by fuel oxidation in the combustion chamber?

A. $C_V(T_4 - T_1)$
B. $C_V(T_3 - T_2)$
C. $C_P(T_4 - T_1)$
D. $C_V(T_4 - T_2)$

16. Which of the following may result from engineering a piston with a high compression ratio?

I. Improved engine efficiency
II. Autoignition of the fuel-vapor mixture prior to sparking
III. A decrease in fuel-vapor density prior to sparking

A. only I
B. only II
C. I and II
D. I, II and III

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First Law of Thermodynamics
Answers and Explanations

1. D
Internal energy is a function of the state of the system. The difference in internal energy between two states is independent of the path taken between them.

2. A
Internal energy change results from the combination of heat flow and work between the system and its surroundings. Conservation of energy determines that the change in internal energy must equal the energy transfered between the system and its surroundings in terms of heat flow and thermodynamic work (pressure-volume work).

$$\Delta U = Q - W$$
$$= Q - P^{*}\Delta V$$

ΔU = internal energy change
Q = heat flow
W = macroscopic work
P^{*} = constant pressure
ΔV = volume change

Internal energy is a function of the state of the system. We were told that the temperature increased somewhat between the initial and final state. For an ideal gas, internal energy depends on the temperature, so we know that internal energy increased.

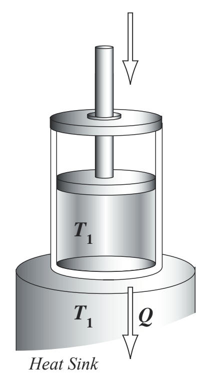
$$U = \frac{3}{2}nRT$$

In summary, the internal energy increase is the same for all three path shown. However, the combination of heat flow and thermodynamic work will be different for each path. On the pressure-volume curve, as the system moves to a greater volume, the work the system performs on the surroundings equals the area under the curve. It is easy to see that the system performed more work along path A. The expansion took place at higher pressures overall so the summation of work performed is greater as reflected in the greater area under the curve. Performing work on the surroundings *costs* the system energy, so if the system performed more work than the other paths but ended up with the same increased final internal energy, then more heat flow, Q, had to occur into the system during the expansion along path A than the other two, and, likewise, more heat flow into the system along path B than C.

3. B
An isothermal process takes place at constant temperature. For an ideal gas, internal energy exists only in the form of the kinetic energy of the particles, so if the temperature does not change for an ideal gas, the internal energy is constant.

$$U = \frac{3}{2}nRT$$

In an isothermal compression, energy added to the system due to work (pressure-volume work) simultaneously departs from the system as heat flow. Likewise, in an isothermal expansion, energy expended by the system in work is constantly replenished by heat flow into the system.

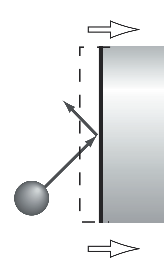


$$\Delta U = 0$$
$$Q = -W$$

First law of thermodynamics for an isothermal process

ΔU = internal energy change
Q = heat flow
W = macroscopic work

4. C
A molecule colliding with the piston head is colliding with a surface moving away from it. This means that while the collision occurs the molecule exerts a force over a distance, performing work ($W = (F \cos \theta)s$). The molecule performs work on the piston head and loses kinetic energy ($W = K_f - K_i = \Delta K$) and thus speed.



5. C
An isothermal transformation is one with constant temperature. If the temperature is constant the pressure-volume product is constant.

$$PV = nRT$$

$$PV = \text{constant}$$

If the temperature is constant the pressure-volume product is constant. All three of the graphs correspond to a constant pressure-volume product except for C which shows volume decreasing exponentially with increased pressure.

6. B

The first law of thermodynamics tells us that if the internal energy of a system changes, there has to be exchange of energy with the surroundings. Some combination of heat flow and thermodynamic work (pressure-volume) work occurred.

$$\Delta U = Q - W$$

In this question our starting point is the heat flow into the system, the heat of vaporization accompanying the transformation of liquid water into waver vapor, which we're asked to relate to the internal energy change. We can rearrange the 1st law to focus on the heat flow, Q.

$$Q = \Delta U + W$$

As heat flows into the liquid water and it transforms into water vapor, water molecules separate from one another along lines intermolecular force increasing in electrostatic potential energy. The water molecules have escaped from the potential energy wells into which they had fallen in the liquid state. Internal energy has increased.

But does the amount of internal energy increase equal the heat flow? That depends on whether or not the system's volume changed and thermodynamic work was performed. If a mole of water transformed into water vapor at 373K, the volume of the system expanded from .018L (the volume of 18g H_2O) to approximately 30L (the volume of a mole of gas at 100°C). The system had to perform work = $P\Delta V$ to push the atmosphere back! In other words, more heat had to flow into the system than simply to bring about the internal energy increase because energy was also required in the work of expanding against the surroundings.

$$\overset{+}{Q} = \overset{+}{\Delta U} + \overset{+}{W}$$

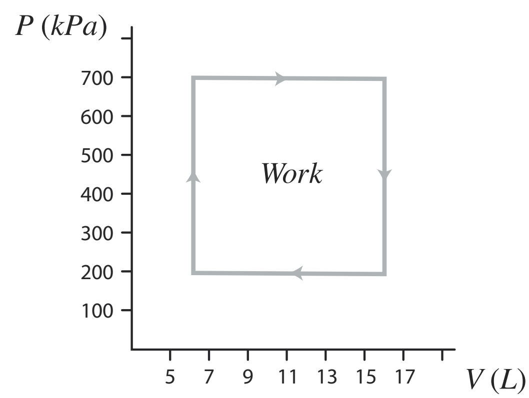
7. D

The question stem uses a bit of thermodynamics nomenclature you should be familiar with. An 'isochoric' transformation is constant volume. An 'isobaric' transformation is constant pressure.

We know that internal energy has no change over the entire cycle because the system returns to its initial state. If the internal energy change is zero, then the exchanges with the surroundings in terms of heat flow must be balanced by thermodynamic work. The engine took heat in, Q_{in}, during the isochoric heating and isobaric expansion stages and expelled heat, Q_{out}, during the isochoric cooling and isobaric compression stages. In order for $\Delta U = 0$, the difference between the heat taken in by this engine and the heat that the engine expels each cycle must equal the work performed.

$$W = Q_{in} - Q_{out}$$

The work performed each cycle equals the area enclosed by the pressure-volume curve.



$$W = (500\text{ kPa})(10\text{ L})$$

$$W = (5 \times 10^5\text{ Pa})(1 \times 10^{-2}\text{ m}^3)$$

$$W = 5000\text{ J}$$

$$Q_{in} - Q_{out} = 5000\text{ J}$$

8. A

An isovolumetric process, also called isochoric, is a thermodynamic process during which the volume of the system remains constant. Because the volume is constant, no pressure-volume work is performed. In terms of the first law of thermodynamics, this means that the internal energy change will exactly equal the heat flow.

$\Delta U = Q - W$	*First law of thermodynamics*
$W = 0$	*Isovolumetric - no work is performed*
$\Delta U = Q$	*First law for an isovolumetric process*

In chemistry, we almost always talk about heat flow, Q, in terms of enthalpy change, ΔH. H is a state function whose change, ΔH, equals Q as long as pressure is constant. So why it is that the heat flow and internal energy change are so often equal in biochemistry?

Heat flow and internal energy change are often equal with biochemical reactions because the reagents and products are usually dissolved. The volume of the chemical system doesn't change with the reaction. Chemical reactions where the work function is significant are typically those with gaseous reagents or products, not dissolved solutes.

One of the most important kinds of intuition for chemistry is moving from the sense of the internal energy change in a reaction to the sense of whether the reaction is exothermic or endothermic and thus likely to be spontaneous or not. Most of the time in biochemistry this conceptual movement isn't going to be complicated by the work function. That's a good thing. If heat flow is happening you know it's because the internal energy changed and vice versa.

9. A

An adiabatic process occurs without any heat flow entering or leaving the system. If there is no heat flow, then any work performed on or by the system must directly correspond to internal energy change.

$\Delta U = Q - W$	*First law of thermodynamics*
$Q = 0$	*'Adiabatic' means no heat flow*
$\Delta U = -W$	*The First law for an adiabatic process*

In other words, the helium in the piston increased in temperature because the surroundings performed work on the gas in the compression, and because the compression was adiabatic all of the energy input remains in the system. No heat flow occurred.

We could use calculus to integrate $P\Delta V$ through the incremental changes in pressure and volume to compute the work. That would be crazy! We have a much easier way to determine how much energy was added to the system. We could imagine a situation where the increase in internal energy *had* occurred by heat flow. It's not too hard to figure out how much heat flow is required to raise the temperature of 1.1L of helium at standard pressure by 40K (ΔT = 313K - 273K = 40K)? The question stem helpfully tells us that the molar heat capacity of helium is 12.5 J mol^{-1} K^{-1}.

$$Q = n\,C\,\Delta T$$

Q	=	heat flow
n	=	number of moles
C	=	molar heat capacity
ΔT	=	temperature change

One mole of an ideal gas at STP occupies 22.4 L. How many moles are 1.1 L of helium at STP?

$$\left(\frac{\text{mol}}{22.4\text{ L}}\right)(1.1\text{ L}) = 0.05\text{ mol}$$

How much heat flow would be required to raise this much helium gas by 40K?

$$Q = (0.05\text{ mol})\left(12.5\ \frac{\text{J}}{\text{mol K}}\right)(40\text{ K})$$

$$= 25\text{ J}$$

That's the answer. Whether the internal energy increase occurred by means of heat flow into the system or by thermodynamic work, we can determine how much of an increase occurred through the change in temperature.

10. A

The 1st law tells us that if the internal energy of a system changes, there has to be exchange of energy with the surroundings. Some combination of heat flow and thermodynamic work (pressure-volume) work occurs.

$$\Delta U = Q - W$$

Let's rearrange this to focus on the heat flow, Q.

$$Q = \Delta U + W$$

The internal energy of the system decreases in the reaction. Oxygen is gaining electrons after all. However, the volume of the system also decreases. We're going from 3 moles of gas at STP to 2 moles of gas at STP, so the system loses 22.4 L of volume. What this means is that some of the heat flow from the system derives from the internal energy decrease and some of the heat flow from the system is due to the work being done on the system.

$$\bar{Q} = \Delta\bar{U} + \bar{W}$$

However, when the reaction is carried out at constant volume, there is no thermodynamic work. In that case the heat flow due to the compression won't be present. Only heat flow due to the original internal energy decrease occurs. At this stage we know enough to choose 'A' because that's the only choice having the reaction occurring in the bomb calorimeter as *less* exothermic.

If we want to be sure, we can determine the missing work. Before determining the thermodynamic work in the compression, remember to convert to SI units.

$$W = P\Delta V$$

$$W = (2.2 \times 10^{-2}\,\text{m}^3)(1 \times 10^5\,\text{Pa}) = 2.2 \times 10^3\,\text{J}$$

$$= 2.2\,\text{kJ}$$

11. D

The intake stroke is the isobaric expansion from state 5 to state 1. The exhaust stroke is the isobaric compression from state 1 to state 5. The work performed by the system (the area under the curve) for the intake stroke is exactly equal and opposite to the work performed on the system in the exhaust stroke.

12. B

This question is about whether you kept your footing in the passage enough to use one of the given formulas. The thermal efficiency is the percentage of the thermal energy produced in the combustion reaction that can be transformed into useful work. The passage presents the thermal efficiency as a function of compression ratio and heat capacity ratio.

$$\eta_{th} = 1 - \frac{1}{r^{\lambda - 1}}$$

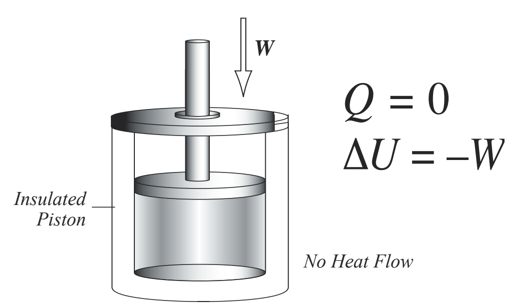
$$\eta_{th} = 1 - \frac{1}{8^{(1.33 - 1)}}$$

$$\eta_{th} = 1 - \frac{1}{8^{\frac{1}{3}}}$$

$$\eta_{th} = 1 - \frac{1}{2} = 50\%$$

13. A

The passage describes the compression stroke from state 1 to state 2 on the Otto cycle diagram as an adiabatic compression. No heat flows in an adiabatic compression, yet temperature increases due to the work performed on the system.



14. D

The 1st law tells us that if the internal energy of a system changes, there has to be exchange of energy with the surroundings. Some combination of heat flow and thermodynamic work (pressure-volume) work occurs.

$$\Delta U = Q - W$$

Let's rearrange this to focus on the heat flow, Q.

$$Q = \Delta U + W$$

In the combustion reaction, internal energy decreases through oxidation-reduction. However, the volume increases as the reaction is carried out at standard conditions (constant P of 1 atm) because the moles of CO_2 and H_2O gas produced are always greater than the moles of O_2 gas consumed. Because the system expands, the heat flow evolved will be less by the work performed than the internal energy decrease.

$$\overset{-}{Q} = \overset{-}{\Delta U} + \overset{+}{W}$$

In the Otto cycle, though, combustion occurs isochorically (constant volume), the transformation from state 2 to state 3 on the diagram. Because no energy is lost to the work of expanding the system, the thermal energy introduced into the piston will be greater per mole of fuel than the heat of combustion of the fuel.

$$\overset{-}{Q} = \overset{-}{\Delta U}$$

When the volume doesn't change, the internal energy change exactly equals the heat flow.

15. B

This is a reading comprehension/association question. It depends on recognition that combustion produces the change from state 2 and state 3 on the Otto diagram. It also helps to understand that the product of a heat capacity and a temperature change is an increase in thermal energy.

16. C

As discussed in the passage, the compression ratio $r = V_1/V_2$, is the ratio of the volume of the piston at state 1 after expansion to the volume at state 2 after compression.

Choice **I** is correct. A higher compression ratio leads to greater efficiency as shown the following formula given in the passage (Note that as described in the passage λ must be a number greater than 1).

$$\eta_{th} = 1 - \frac{1}{(V_1/V_2)^{\lambda-1}} = 1 - \frac{1}{r^{\lambda-1}}$$

Choice **II** is also correct. A high compression ratio is consistent with a low volume at state 2 after adiabatic compression. No heat flows in an adiabatic compression. The temperature increases due to the work performed on the system. Compression to a lower volume entails more work performed on the piston and a greater temperature of the fuel vapors prior to ignition. If the temperature after compression exceeds the autoignition temperature of the fuel vapors they will ignite without the need for a spark. This is exactly what happens in a diesel engine which has no spark plugs. Diesel engines are engineered with a higher compression ratio than gasoline engines.

Choice **III** is incorrect. Lower fuel vapor density prior to sparking is consistent with a higher volume at state 2. All else equal, a higher volume at state 2 would produce a lower compression ratio not a higher compression ratio.

Second Law of Thermodynamics Practice Items

1. When a hot stone is dropped into a cool water bath and heat flows from the stone into the bath

A. More entropy is lost in the stone than gained by the water.
B. More entropy is gained by the stone than lost by the water.
C. Less entropy is lost by the stone than gained by the water.
D. The change in entropy in the stone is balanced by an equal and opposite change in entropy in the water.

2. Free expansion is an irreversible process in which a gas expands into an insulated evacuated chamber. During a free expansion of an ideal gas

I. the temperature remains constant.
II. the entropy of the gas increases.
III. the internal energy of the gas remains constant.

A. I
B. I and III
C. II and III
D. I, II, and III

3. Which of the following statements is NOT true?

A. The Carnot cycle is reversible.
B. The entropy of the universe has increased after a complete Carnot cycle.
C. The Carnot cycle is never 100% efficient.
D. all of the above are true

4. Using the key below, which of the following sequences represents either a forward or reverse Carnot cycle?

AA. adiabatic expansion
BB. adiabatic compression
CC. isothermal expansion
DD. isothermal compression

A. DD, AA, CC, BB
B. CC, AA, DD, BB
C. BB, DD, AA, CC
D. more than one of the above

5. Which of the following does NOT change for an ideal gas undergoing adiabatic compression?

A. entropy
B. internal energy
C. pressure
D. volume

6. Which of the following does NOT change for an ideal gas undergoing isothermal compression?

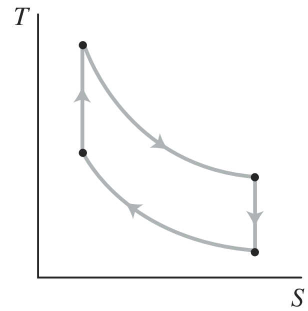
A. entropy
B. internal energy
C. pressure
D. volume

7. What is the maximum efficiency of an engine operating between 177 ^{o}C and 27 ^{o}C?

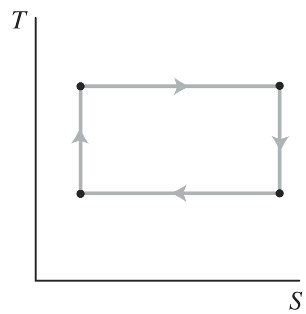
A. 15%
B. 33%
C. 50%
D. 85%

8. Which of the following graphs represents temperature vs. entropy of the ideal gas serving as the working fluid within a Carnot engine operating in the forward direction?

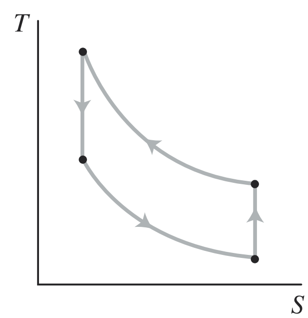
A.



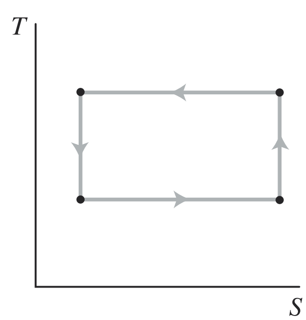
B.



C.



D.



9. A real-world heat pump in a sun room has a coefficient of performance of 3.0 when the interior temperature is 23°C and the outside temperature is 2°C. How much electrical power is consumed for it to produce 480 watts of heating in these conditions?

A. 34 W
B. 63 W
C. 160 W
D. 480 W

10. A sealed container holds 1-L of hydrogen gas (H_2) at STP. A second sealed container holds 1-L of helium gas (He) at STP. Both containers are heated isochorically to 100°C. Which gas experiences the greatest change in entropy?

A. the hydrogen gas
B. the helium gas
C. both have equal changes in entropy
D. the entropy of neither gas changes

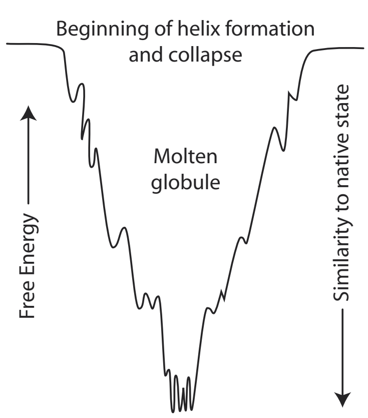
11. What is the relationship between a microstate and a macrostate of a thermodynamic system?

A. A microstate is a point in phase space, whereas a macrostate describes the relationship of the system to its surroundings.
B. A microstate corresponds to the quantum electrodynamic parameters of the system, whereas a macrostate describes the system as measured in the laboratory.
C. A macrostate corresponds to the macroscopic properties of a statistical ensemble comprised of the accessible microstates.
D. A microstate is defined by specifying external parameters whereas a macrostate is defined as a state for which particle motions are completely specified.

Passage (Questions 12-16)

Conformational entropy is the entropy associated with the physical arrangement of a polymer chain that assumes a compact or globular state in solution. The concept is most commonly applied to biological macromolecules such as proteins and RNA, but can also be used for polysaccharides and other polymeric organic compounds. To calculate the conformational entropy, the possible conformations assumed by the polymer may first be discretized into a finite number of states, usually characterized by unique combinations of certain structural parameters, each of which has been assigned an energy level. In proteins, backbone dihedral angles and side chain rotamers are commonly used as descriptors, and in RNA the base pairing pattern is used. These characteristics are used to define the degrees of freedom in the statistical mechanics sense of a possible microstate. The conformational entropy associated with a particular conformation is then dependent on the probability associated with the occupancy of that state.

The folding funnel hypothesis is a specific version of the energy landscape theory of protein folding, which assumes that a protein's native state corresponds to the global free energy minimum under the solution conditions usually encountered in cells. Although energy landscapes may be "rough", with many non-native local minima in which partially folded proteins can become trapped, the folding funnel hypothesis assumes that the native state is a deep free energy minimum with steep walls, corresponding to a single well-defined tertiary structure.



In the canonical depiction of the folding funnel, the depth of the well represents the energetic stabilization of the native state versus the denatured state, and the width of the well represents the number of possible conformations assumed by the protein. The surface outside the well is shown as relatively flat to represent the heterogeneity of the random coil state.

The entropy of heterogeneous random coil or denatured proteins is significantly higher than that of the folded native state. In particular, the conformational entropy of the amino acid side chains in a protein is thought to be a major contributor to the stabilization of the denatured state and thus a barrier to protein folding. However, a recent study has shown that side-chain conformational entropy can stabilize native structures among alternative compact structures. The conformational entropy of RNA and proteins can be estimated; for example, empirical methods to estimate the loss of conformational entropy in a particular side chain on incorporation into a folded protein can roughly predict the effects of particular point mutations in a protein. Side-chain conformational entropies can be defined as Boltzmann sampling over all possible rotameric states. If a system can be in states n with probabilities P_n

$$S = -k_B \sum_{n=1}^{W} P_n \ln(P_n)$$

where k_B is Boltzmann's constant and W is the number of possible conformations.

It has been proposed that decreasing the conformational flexibility of the unfolded chain (by substitution with proline, or by replacement of glycine) should lead to an increase in the stability of the folded relative to the unfolded protein The limited conformational range of proline residues lowers the conformational entropy of the denatured state and thus increases the free energy difference between the denatured and native states. A correlation has been observed between the thermostability of a protein and its proline residue content.

12. When the temperature of a dilute aqueous solution containing globular proteins is increased from 29°C to 32°C, the conformational entropy of the suspended polypeptides

- **A.** decreases
- **B.** remains the same
- **C.** increases
- **D.** becomes less predictable

13. Based on the information presented in the passage, for a protein complex with a ligand-binding interface, point mutations downstream from the binding site that increase conformational entropy are most likely

- **A.** to decrease protein-ligand binding.
- **B.** to increase protein-ligand binding.
- **C.** not to alter protein-ligand binding.
- **D.** to increase catalytic efficiency.

14. In the folding funnel hypothesis, the width of the well in the diagram increases with

- **A.** conformational entropy
- **B.** free energy
- **C.** intramolecular hydrogen bonding
- **D.** protein solubilization

15. A theoretical polymer with only one possible conformation would

- **A.** have zero conformational entropy.
- **B.** possess conformational entropy equal to k_B.
- **C.** be at absolute zero, 0-K, temperature.
- **D.** possesses conformational entropy due to possible states of translational momentum.

16. A mutation substituting two proline residues downstream from the active site of an enzyme reduced enzyme activity. These substitutions also eliminated a hydrogen bond and some hydrophobic interactions suggesting that

- **A.** those factors outweighed the decrease in conformational stability.
- **B.** the substitutions increased the conformational entropy of the native protein.
- **C.** these factors altered the shape of the binding site.
- **D.** the conformational flexibility of the unfolded chain was increased.

Second Law of Thermodynamics
Answers and Explanations

1. C

The temperatures of the stone and bath are changing as heat flows, so the mathematical expression of the idea would be more complicated than below, but to get the basic idea we can look to the entropy change for heat flow in a reversible process. The higher the temperature, the less entropy change occurs with a given amount of heat flow.

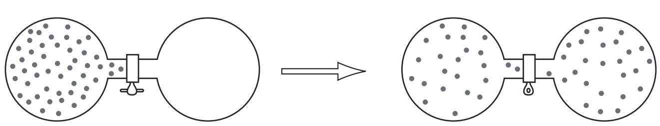
$$\Delta S = \frac{Q_r}{T}$$

ΔS = entropy change
Q_r = heat flow (in reversible process)
T = temperature

As heat flow occurs from the hot stone into the cold water, the water gains entropy and the stone loses entropy as thermal energy increases and decreases respectively. However, because the stone is a higher temperature than the water, as predicted in the formula above, the amount of entropy lost by the stone for the same amount of heat flow is less than the amount of entropy gained by the water. Put it together and the entropy of the universe is increasing, which is why the heat flow is spontaneous.

2. D

Free expansion can be observed in the apparatus below when the stopcock is opened between two bulbs connected by a tube. In the initial state, all of the gas (N particles) is constrained to occupy the bulb on the left. The bulb on the right contains a vacuum. When the stopcock is opened, the gas spontaneously moves to occupy both bulbs.



Choice **I** and III are correct for the same reason. No heat flow occurs, and no work is done when the stopcock is opened and the gas expands to fill the vacuum. If there is no heat flow and no thermodynamic work there is no internal energy change.

$$\Delta U = Q - W$$

And with an ideal gas where thermal energy is the only form of internal energy, if there is no internal energy change, the temperature must remain constant.

$$U = \frac{3}{2}nRT$$

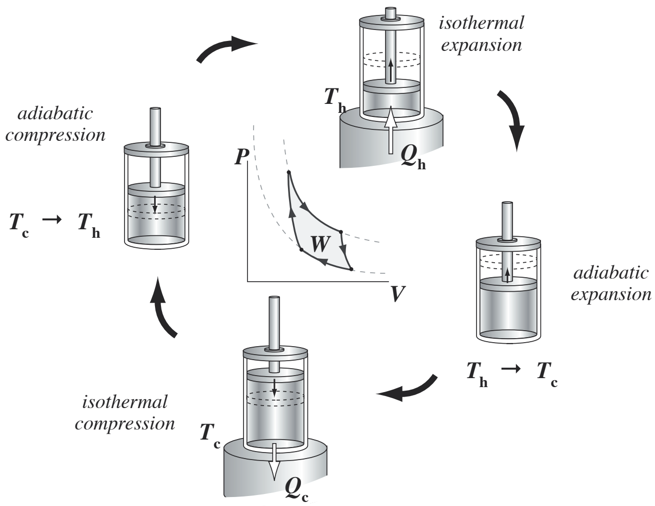
Choice **II** is also correct. In the initial state, all of the gas (N particles) is constrained to occupy a single bulb. When the stopcock is opened, the gas spontaneously moves to occupy both bulbs. In this example, with the stopcock opened, the probability of the second state is 2^N times that of the initial state. The entropy has increased proportional to the logarithm of this increased probability.

3. B

The Carnot cycle consists of four stages: an isothermal expansion on the hot sink, an adiabatic expansion, an isothermal compression on the cold sink, and lastly an adiabatic compression. Each of these stages is reversible. Because each stage individually doesn't increase the entropy of the universe, neither does the complete cycle. The Carnot cycle is the ideal theoretical cycle. It does not increase the entropy of the universe. The Carnot cycle allows us to define the minimum amount of heat which must be delivered to the cold sink, in other words, the amount of heat that isn't available to be converted into work. The Carnot cycle represents the boundary between possible and impossible engines, between a spontaneous engine that runs forward and increases the entropy of the universe and an impossible engine that would cause the entropy of the universe to decrease.

4. D

All three choices represent either a forward or reverse Carnot cycle. (In the reverse Carnot cycle, work is done to extract heat from the cold sink and expel it into the hot sink.)



5. A

The entropy doesn't change for the system through an adiabatic expansion or compression. Although the temperature does increase, for example, in the case of an adiabatic compression, the volume decreases, and the two effects balance each other with respect to entropy change. An adiabatic compression is isentropic, in other words.

6. B

An isothermal compression is one in which the temperature does not change. The internal energy is only thermal energy, ie. the kinetic energy of the particles, with an ideal gas, so the internal energy goes up or down with the temperature. If the temperature is constant so is the internal energy.

$$U = \frac{3}{2}nRT$$

It should be noted that even though the entropy of the gas decreases as heat flows out into the surroundings during an isothermal compression and its volume decreases, the surroundings experience an entropy increase of exactly the same magnitude, being the same temperature, so even though the entropy of the system changes in an isothermal compression, the entropy of the universe does not change, so like the adiabatic compression, an isothermal compression is also reversible.

7. B

The Carnot efficiency describes the maximum possible amount of heat input that can be transformed into work for a given engine.

$$\varepsilon = \frac{W}{Q_h} = 1 - \frac{T_c}{T_h}$$

A key with this problem is to remember to convert the temperature values from Celsius to Kelvin (K = °C + 273).

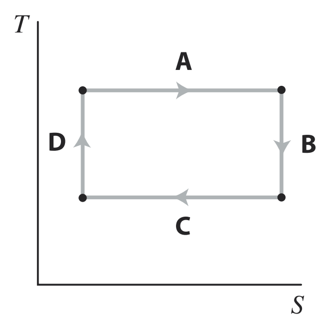
$$T_c = 27°\text{C} = 300\text{ K}$$

$$T_h = 177°\text{C} = 450\text{ K}$$

$$\varepsilon = 1 - \frac{300\text{ K}}{450\text{ K}} = 33\%$$

8. B

Each stage on the figure below represents a stage in the forward Carnot cycle.



A – Isothermal expansion on the hot sink
B – Adiabatic expansion
C – Isothermal compression on the cold sink
D – Adiabatic compression

9. C

A heat pump is a heat engine operating in reverse. A heat pump applies work to extract heat, Q_c, from a cold sink and delivering a larger amount of heat, Q_h, to a hot sink. ($Q_h = Q_c + W$). The coefficient of performance, COP, of a heat pump is the ratio of the heat delivered to the hot sink to the work required to transfer the heat.

$$\text{COP} = \frac{Q_h}{W}$$

W is the energy purchased from the power company. Q_h is the heating delivered to the home.

$$3 = \frac{480\text{ W}}{W}$$

$$W = 160\text{ W}$$

To deliver 480 W of heating, the homeowner must supply 160 W of electrical power to his real world heat pump.

Note that if the heat pump were working ideal Carnot efficiency, its COP would be greater. We didn't need the given temperatures to answer the particular question asked, but after converting them into Kelvin (K = °C + 273), we can determine the Carnot COP.

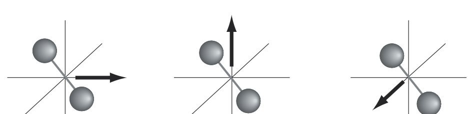
$$\text{COP}_{\text{Carnot}} = \frac{T_h}{T_h - T_c}$$

$$= \frac{296\text{K}}{296\text{K} - 275\text{K}}$$

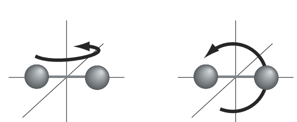
$$= 14.1$$

10. A

With the ability to store energy in both vibrational and rotational modes, a diatomic gas like H_2 has more partitions for thermal energy. As a sample of diatomic gas takes in heat, the energy spreads out into all of its degrees of freedom. H_2 has a higher molar heat capacity than He.



translational motion - three degrees of freedom



rotational motion - two degrees of freedom

vibration - two degrees of freedom

Because it has a higher molar heat capacity, more heat flow into the H_2 sample will occur than into He to obtain the same temperature change. Although the expression demonstrating the entropy change over the temperature change would require calculus, it's easy to see that it would be greater for H_2. A greater amount of heat flow corresponds to a greater entropy change.

$$\Delta S = \frac{Q_r}{T}$$

ΔS = entropy change
Q_r = heat flow (in reversible process)
T = temperature

Another way to understand why H_2 will have greater entropy at a given temperature than He is visualized in the degrees of freedom pictured above. Because H_2 has rotational and vibrational degrees of freedom in addition to translational, at a given temperature the macrostate of H_2 supervenes on a much larger ensemble of microstates. There are a great many more combinations for the distribution of thermal energy with H_2 among its degrees of freedom than with He.

11. C

A microstate is a specific microscopic configuration of a thermodynamic system that the system may occupy with a certain probability in the course of its thermal fluctuations. Think of a microstate as a snapshot of every particle at a moment in time. In contrast, the macrostate of a system refers to its macroscopic properties, such as its pressure, volume, and temperature. Microstates appear as different possible ways the system can achieve a particular macrostate. In other words, a macrostate is characterized by a probability distribution of possible states across a certain statistical ensemble of all microstates.

Entropy rises with the multiplicity of the system (the number of possible microstates that correspond to a particular macrostate).

$$S = k \ln X$$

S = entropy
k = Boltzmann's constant
X = multiplicity

12. C

The vibrational and rotational kinetic energy of the various groups within the protein increase with temperature, and the conformational space explored grows. As the protein transitions to the unfolded state the number of conformations underlying the state of the protein increases exponentially.

13. A

Although the passage mentions the possibility of exceptions, the passage makes the arguments and presents evidence supporting the conclusion that increases in conformational flexibility in distant groups will stabilize an unfolded protein. This will decrease the possibility of an ordered binding site, so the changes proposed should lead to a decrease in protein-ligand binding affinity.

14. A

The passage describes the width of the well as representing the number of possible conformations assumed by the protein. The conformational entropy of a state increases with the number of conformations underlying it.

15. A

Conformational entropy is defined in the passage as the entropy associated with the physical arrangement of the polymer chain. If there is only one possible configuration, the conformational entropy would be zero. The entropy value returned by Boltzmann sampling as described by the formula in the passage would be zero (ln(1) = 0).

$$S = -k_{\mathrm{B}} \sum_{\mathrm{n}=1}^{\mathrm{W}} P_{\mathrm{n}} \ln(P_{\mathrm{n}})$$

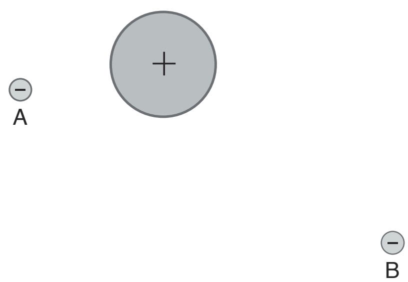
There may be other forms of entropy such as those involving degrees of freedom in translational motion, hydration or crystal packing, for example, but conformational entropy would be zero.

16. C

The discussion in the passage of the effects of proline substitutions to stabilize the native state of a globular protein were focused on the decrease in the conformational entropy of the unfolded state. Changes that increase the enthalpy of the native state, even if they decrease the entropy of the unfolded state, might make an ordered active site less likely and decrease enzyme activity.

Electricity Practice Items

1. Two negative point charges, A and B, are located in the space around a uniform sphere (radius r_B) of positive charge density.



Charge A has magnitude $-Q_o$ and is located at a distance $2r_B$ from the edge of the sphere. Charge B has magnitude $-2Q_o$ and is located at a distance $5r_B$ from the edge of the sphere. What is the ratio of the magnitude of the electrostatic force exerted by the sphere upon A to the magnitude of the force it exerts upon B?

A. 25:8
B. 2:1
C. 5:4
D. 1:1

2. A storm-cloud can be thought of as one plate of a giant capacitor, with the earth being the other plate. With which of the following units would we measure the charge contained in a storm-cloud?

A. coulombs
B. farads
C. amperes
D. volts

3. A volt is a

A. joule per coulomb
B. ampere · ohm
C. electron volt per electron
D. all of the above

4. A negatively charged rubber rod, without touching, is brought in close proximity to an electrically neutral metallic sphere. The sphere has a copper wire running from its base into a large copper plate buried in moist soil. The wire is cut and the rubber rod is removed from the proximity of the sphere. Which of following describes the present condition of the sphere?

I. The surface of the sphere is negatively charged.
II. The surface of the sphere is positively charged.
III. The potential difference between the ground and the sphere is positive.

A. I
B. II
C. I and III
D. II and III

5. Which of the following would serve as a poor conductor?

A. molten sodium chloride
B. metallic silver
C. fused quartz (SiO_2)
D. all are good conductors

6. What is the electric field intensity at a point 30 millimeters from a charge of 1×10^{-5} C? (Coulomb constant $k = 9 \times 10^{9}$ N m^2 C^{-1})

A. 1.0 N/C
B. 1.0×10^{6} N/C
C. 3.0×10^{6} N/C
D. 1.0×10^{8} N/C

7. After passing through the aperture of the electron gun, a cathode ray travels parallel to an electric field moving from a zone of high potential towards an area of lower potential. As the electrons move from the higher potential towards the lower potential area

A. they slow down.
B. their potential energy decreases.
C. the path of the cathode ray curves.
D. they maintain a constant velocity.

8. The molar heat capacity of helium is very close to that of an ideal gas (3/2 R). One mole of protons contains a magnitude of charge equal to 96,500 C (1 faraday). Approximately how much will the temperature rise in one mole of alpha particles (He^{2+}) moving in a vacuum in through a 1 mV decrease in potential?

A. -8 K
B. 8 K
C. 15 K
D. 31 K

9. Two point charges, one with a charge of +90.0μC and the other with a charge of −10.0μC are placed 10 mm apart. At what point along the axis between them is the electric field zero?

A. 1.0mm from the +90.0μC charge
B. 2.5mm from the +90.0μC charge
C. 2.5mm from the −90.0μC charge
D. at no point on the axis between them

10. Introducing a dielectric substance between two parallel charged plates

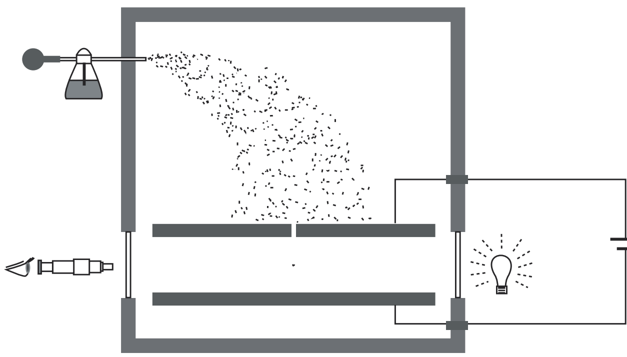
A. weakens the electric field between the plates.
B. increases the potential difference between the plates.
C. creates a uniform electric field in the space between the plates.
D. decreases the capacitance of the plates.

11. Which of the following occurs with an increase in electrostatic potential energy?

A. A gaseous sodium ion captures an electron.
B. Negative charges introduced at a point on a neutral metal sphere spread over its surface area with uniform distribution.
C. One glucose molecule reacts with six molecules of oxygen to form six molecules of carbon dioxide and six molecules of water.
D. A globular polypeptide unfolds from its native configuration in high temperature conditions.

Passage (Questions 11-16)

In 1909, the U.S. physicist Robert Millikan (1868-1953) performed a series of experiments, in which, by observing the behavior of electrically charged oil droplets within a uniform electric field, he was able to determine the charge on an electron. The apparatus utilized by Millikan in these experiments is illustrated by the schematic below. A spray bottle produces a fine mist of oil droplets. Some of these pass through an aperture into a viewing chamber, where they may be observed.



To determine the mass of a particular droplet, the experimenter observes the rate at which it falls in the earth's gravitational field. The drop reaches a terminal velocity, which depends on the mass of the droplet, the oil density and the viscosity of the air.

Some of the oil droplets possess an electric charge, acquired when they attach themselves to ions produced by the irradiation of the surrounding air with X-rays. When such a droplet passes into the viewing chamber, the electric field of the two charged plates produces a force on the droplet opposite in direction to that of the earth's gravitational field. If one adjusts the voltage on the plates, the electrical force on the droplet can be made to balance the gravitational force exactly, and the droplet will remain suspended.

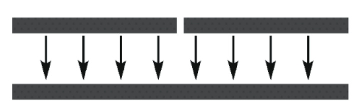
In this manner, the experimenter determines the magnitude of the electric field necessary to produce a force on the droplet equal in magnitude to the opposing force of gravity. The charge on the droplet may then be determined. Millikan obtained values such as the following for the magnitude of the charge on certain droplets:

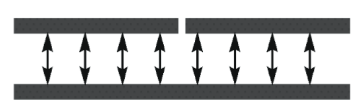
$$q_1 = -1.6 \times 10^{-19}\text{ C}$$
$$q_2 = -6.4 \times 10^{-19}\text{ C}$$
$$q_3 = -8.0 \times 10^{-19}\text{ C}$$

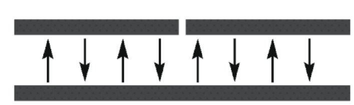
12. If the electrical field strength is of slightly lower magnitude than necessary to oppose the action of the earth's gravitational field, the oil droplet slowly descends. As it does so

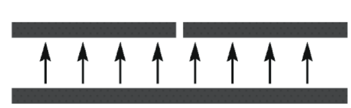
A. both its electrostatic potential energy and gravitational potential energy decrease.
B. its electrostatic potential energy decreases while its gravitational potential energy increases.
C. its electrostatic potential energy increases while its gravitational potential energy decreases.
D. both its electrostatic potential energy and gravitational potential energy increase.

13. Which of the following is the most proper representation of the electric field between the charged plates?

A. 

B. 

C. 

D. 

14. If the charged plates in the Millikan apparatus, separated by a distance of two centimeters, have a potential difference of 1200 volts, what is the magnitude of the electric field between the plates away from the edges of the plates?

A. 2,400 N/C
B. 60,000 N/C
C. 240,000 N/C
D. 480,000 N/C

15. Which of the following statements are true with regard to the drag force of the air upon the droplet at terminal velocity and the electrical force upon the droplet at the later time it is suspended within the viewing chamber?

I. The two forces have equal magnitude.
II. Both forces are conservative.
III. Neither force performs work.

A. I only
B. I and III
C. II and III
D. I, II, and III

16. The charges on the oil droplets possess magnitudes commonly divisible by 1.6×10^{-19} C. The fact that electric charge cannot be more finely divided is a consequence of

A. the conservation of electric charge.
B. Coulomb's law.
C. the quantum nature of electric charge.
D. the photoelectric effect.

17. Using q as the charge on a certain droplet, m as its mass, d as the distance between the charged plates, and g as the acceleration due to gravity, express the correct voltage to apply across the plates in order to suspend the droplet in the viewing chamber?

A. $\frac{mgd}{q}$

B. $\frac{mgq}{d}$

C. $\frac{mgq}{d^2}$

D. $\frac{mgd^2}{q}$

Electricity
Answers and Explanations

1. B

The difference between the magnitude of force exerted by the sphere on charge A vs. B is determined by their different distances from the center of the sphere and their respective charges. (A sphere upon which charge has been uniformly spread exerts force just as if its charge were concentrated at its center). The electrostatic force is inversely proportional to the square of the distance between each charge and the center of the sphere and directly proportional to the charge of each. Charge A is located a distance of 3r from the center of the sphere, and charge B is located a distance of 6r from the center of the sphere.

$$F = k\frac{q_1 q_2}{r^2}$$

$$F_A = k\frac{Q_0 q_{sph}}{(3r_A)^2} \qquad F_B = k\frac{2Q_0 q_{sph}}{(6r_A)^2}$$

$$F_A : F_B$$

$$\frac{1}{9} : \frac{1}{18}$$

$$2 : 1$$

2. A

We would express the *capacitance* of the cloud in farads. The capacitance governs the relationship between the amount of charge stored in the cloud and the voltage between the cloud and the ground.

3. D

Each of these is a useful way to express one volt, but choice 'A', is especially helpful for understanding the energy description of an electrostatic system. When you say that the potential difference between two charged plates is 5 V, you are saying that the electric field between the plates can do 5 joules of work on 1 coulomb of charge. It would take 5 joules of work to move a coulomb from one plate to the other against the field. You're saying that a coulomb of positive charge has 5 joules more potential energy near the positive plate. Voltage is about the capability of a field to expend or store energy through work on charge between two locations within the field.

4. D

The negatively charged rod repels electrons in the metal sphere which then travel through the wire into the ground. Upon cutting the wire the sphere will now be positively charged on its surface. Excess charge collects along the surface of a conductor.

The potential difference between points A and B is defined as the change in potential energy per unit charge of a positive charge q moved from A to B. The potential difference between the ground and the sphere is positive. Next to the sphere is a place where a positive test charge would have a positive potential energy.

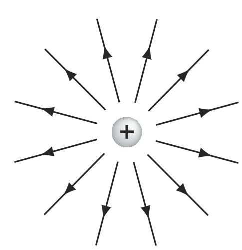
5. C

Both molten sodium chloride and metallic silver possess mobile charge carriers, ions and electrons respectively. They are good conductors. Fused quartz (glass) is a molecular solid. It is a poor conductor.

6. C

The electric field of a point charge represents the capability of that charge to exert force, a capability permeating the space around the charge, and just as the electric force exerted by a point charge decreases with the square of the distance (Coulomb's Law), the electric field intensity decreases with the square of the distance from a point charge

$$E = k\frac{q}{r^2}$$



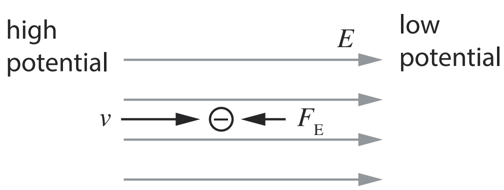
To determine the field intensity at a point 30 mm from a charge of 1×10^{-5} C:

$$E = \frac{(9.0 \times 10^{9}\ \mathrm{N \cdot m^2/C^2})(1 \times 10^{-5}\ \mathrm{C})}{(3 \times 10^{-2}\ \mathrm{m})^2}$$

$$E = 1\ \frac{\mathrm{N}}{\mathrm{C}}$$

7. A

A cathode ray is a beam of electrons. We are told in the question stem that after the electrons pass through the aperture of the electron gun they enter a zone of high potential, traveling towards a zone of low potential parallel to the electric field within the zone they have entered. The figure below shows an electron in a cathode ray traveling parallel to an electric field moving from a zone of high potential towards an area of lower potential.



Electric field lines are always drawn to show the orientation of the force that would be exerted on a *positive* test charge. The force on an electron will be in the opposite direction of the field lines.

From a dynamics/kinematics perspective the force on an electron within this electric field would cause an acceleration opposite the direction of particle velocity, so the electrons will slow down.

From a work and energy perspective, a zone of high potential is a location where a positive charge would have a high potential energy. Conversely, a negative charge has a low potential energy within a high potential zone, and a high potential energy where the electrical potential is low. In other words, as the electrons move through the field their kinetic energy is being transformed into potential energy in much the same way as would occur if you threw a ball straight up against the gravitational field.

8. C

As the α particles move from the higher to lower potential, electrostatic potential energy will be transformed into kinetic energy. To determine the amount of kinetic energy they gain, the first thing we need to do is convert the quantity of α particles given in moles into an SI quantity of electric charge in Coulombs.

$$\left(\frac{96{,}500\ \mathrm{C}}{\mathrm{mol\ e}}\right)\left(\frac{2\ \mathrm{mol\ e}}{\mathrm{mol\ He^{2+}}}\right) = 1.93 \times 10^{5}\ \mathrm{C}$$

The potential difference tells us how many joules per coulomb of work the field does through the potential drop. We can multiply the voltage by the charge to determine how much kinetic energy the α particles gained.

$$1.93 \times 10^{5}\ \mathrm{C}\left(\frac{1 \times 10^{-3}\ \mathrm{J}}{\mathrm{C}}\right) = 1.93 \times 10^{2}\ \mathrm{J}$$

From a thermodynamic perspective, the additional kinetic energy of the particles is a form of thermal energy. We can imagine this new thermal energy as the product of heat flow into the α particles to determine the temperature change.

$$Q = n\,C\,\Delta T$$

$$1.93 \times 10^{2}\ \mathrm{J} = (1\ \mathrm{mol})(12.6\ \mathrm{J\ mol^{-1}\ K^{-1}})\ \Delta T$$

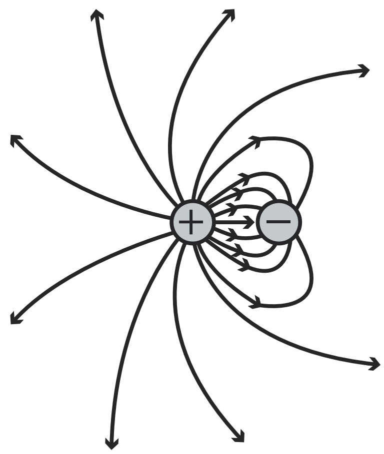
$$\Delta T = 15\ \mathrm{K}$$

Note that the question expects you to know the SI ideal gas constant, $R = 8.3\ \mathrm{J\ mol^{-1}\ K^{-1}}$. It's very difficult to tell the MCAT's disposition about the ideal gas constant . . . Also $R = 0.082\ \mathrm{L\ atm\ mol^{-1}\ K^{-1}}$.

Additionally, because the answers are widely spaced numerically, this is a safe problem for mental math. If instead of 1.93×10^5 you use 2×10^5 and instead of 12.6 for 3/2 R you just use 12, the answer will come out as 16 or 17K, which is okay!

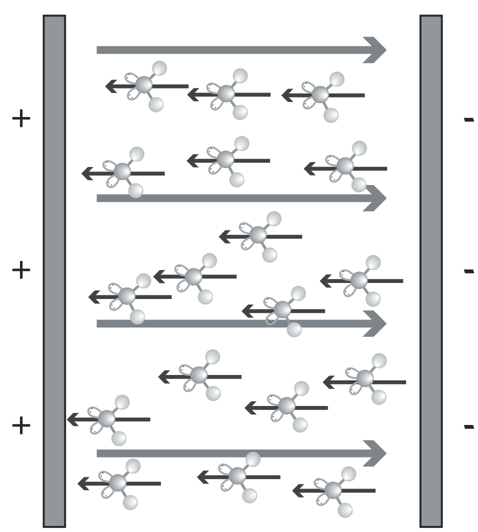
9. D

Two unlike point charges constitute a dipole. The net field of the two charges in the problem is that of a dipole with two unequal charges. Although such a point would exist if these were two like charges, there is no point on the axis between two unlike charges where the net field is zero.



10. A

Water is an example of a dielectric substance. With a dielectric substance the application of an external electric field causes the substance in bulk to polarize in opposition to the applied field.



Water molecules are polar. Under the influence of an external field they rotate with their negative poles towards the positive plate and positive poles towards the negative plate so that so that their dipole moment vectors are in alignment opposite the external field. Between the plates of a parallel plate capacitor the net result is to weaken the net electric field between the plates.

It should be noted that if the same charge densities on the plates now produces a weaker net field between the plates, the voltage across the plates is now lower ($V = Ed$ - the voltage is the product of the strength of the field and the distance of plate separation.) If there is a given charging voltage, this means that the plates can now store more charge before reaching that charging voltage. In other words, the introduction of the dielectric increased the capacitance of the capacitor.

The property of water to behave as a dielectric has consequences so far reaching it suffices to say you should basically always be thinking about it. It goes towards explaining how it is that salts can dissolve in water or how the hydration layer of proteins prevents protein precipitation. It goes towards understanding why voltage gated channels must be much closer on a nonmyelinated neuron than on a nonmyelinated neuron. Lots of important stuff!

11. D

It's a good heuristic approach for interpretation to remember that electrostatic potential energy *increases* in a system of charged particles if like charges are being forced closer together or unlike charges pulled further apart. It takes work to do those things. Imagine moving the charges yourself. Would it take work to do it? If you have two positive charges or two negative charges, it takes work to push them together against electrostatic force of repulsion, so you're storing energy in the system when you do that. Similarly, it takes work to pull a positive and a negative apart. It gets easier as you go, but you're storing energy the whole way.

Conversely, electrostatic potential energy *decreases* when things are happening in the opposite direction. When like charges are moving further apart, potential energy is decreasing, and when unlike charges are falling together, potential energy is also decreasing. Unlike charges fall together down into a well.

This is what's happening with choice 'A'. The sodium ion and the electron come together with a decrease in electrostatic potential energy. It would take work to pull the electron off the neutral atom. When they fall together it's a decrease.
Likewise it's a decrease in choice 'B' when negative charges introduced at a point on a neutral metal sphere spread over its surface area with uniform distribution. Like charges fall away from each other with decrease in electrostatic potential energy.

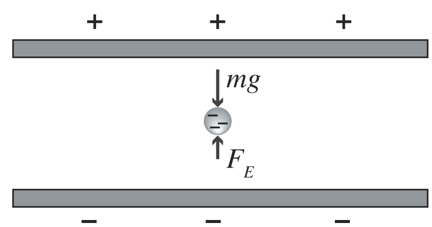
Choice 'C' is more complicated, obviously. We're priming some very important ideas, so don't hold it against yourself if it didn't seem obvious at first this is an electrostatic potential energy decrease. There are a few ways to think about it. Firstly, we know that the combustion occurs with heat flow out of the system. It's exothermic, and this is while the system is expanding. That means by the 1st law of thermodynamics that there must have been an internal energy decrease. ΔU is negative.

What is the form of that internal energy decrease? You have same charged particles on the left of the reaction as on the right, by which we mean carbon, hydrogen and oxygen nuclei and electrons. They have changed their configuration, their relationship in space. Electrons which had been in bonding orbitals between carbon and carbon and carbon and hydrogen are now in bonding orbitals between oxygen and carbon and oxygen and hydrogen. The internal energy decrease is electrostatic potential energy decrease as oxygen pulls the new electrons it has gained access to towards its big nucleus. Another way to say the same thing is that oxygen is oxidizing carbon and oxygen has a big positive reduction potential, so we know electrostatic potential energy is decreasing.

So choice 'D' is the answer. As the polypeptide unfolds, salt bridges come apart. A glutamate pulls away from a lysine. An aspartate pulls away from an arginine. Hydrogen bonds underlying secondary structure pull away from each other as α helices and β pleated sheets become disorganized. Water has to move away from itself to make room for exposed hydrophobic side chains. Across many domains throughout the polypeptide within its surroundings the electrostatic potential energy is increasing.

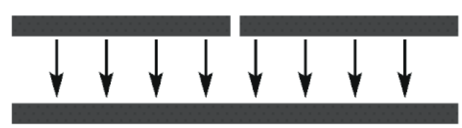
12. C

As the negatively charged oil droplet descends, it's gravitational potential energy decreases. However, it is moving *against* electrostatic force from a zone of positive potential to a zone of negative potential. It's electrostatic potential energy is increasing.



13. A

The best representation is a uniform electric field in the space between the plates with parallel electric field vectors directed downward. The larger the plates and the smaller the distance between them the more uniform the field.



Because electric field lines always show the force they would exert on a positive charge, the field as depicted would exert an *upward* force on a negative charge. We are told in the passage that the electric force is equal and opposite to the weight in the lower chamber, so the electric force must be upward. Millikan's findings report negatively charged droplets, so the field lines must be oriented downward. Additionally, the figure shows the positive plate connecting to the positive terminal of the voltage source which also corresponds to field lines oriented downward.

14. B

The voltage across parallel charged plates is the product of the strength of the uniform electric field between the plates and the distance between the plates.

$$V = Ed$$

$$E = \frac{V}{d}$$

$$E = \frac{1.2 \times 10^{3}\ \mathrm{V}}{2 \times 10^{-2}\ \mathrm{m}}$$

$$E = 6.0 \times 10^{4}\ \frac{\mathrm{V}}{\mathrm{m}}$$

The units of electric field may be expressed as either V/m or N/C. These are equivalent, though each gives you a different way to think about the field.

15. A

There is no net force on the droplet, neither when it is falling at terminal velocity in the upper chamber nor when it is suspended motionless in the lower chamber. In the upper chamber the equal and opposite forces on the droplet are the weight downwards and the frictional drag force upwards. In the lower chamber the equal and opposite forces on the droplet are the weight downwards and the electrostatic force upwards. Because both forces are equal and opposite to the weight, the drag force and the electrostatic force must be equal, so **I** is true.

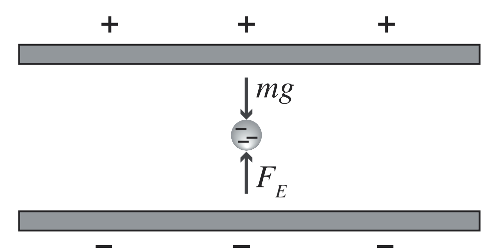
II is not true because friction forces are not conservative. Energy dissipates through friction, and **III** is not true because the friction force performs work (force times distance). As the droplet descends at terminal velocity its decrease in gravitational potential energy will equal the energy dissipated through friction (it's not becoming kinetic energy after all, as the speed is constant).

16. C

The quantum nature of charge means the magnitude of charge must be an integral multiple of *e*. It's actually a bit more complicated. It's not completely correct to say that *e* is the smallest possible charge in nature. In actuality, all known elementary particles, including quarks, have charges that are integer multiples of 1/3 *e*. Therefore, one can say that the "quantum of charge" is 1/3 *e*. . In this case, one says that the "elementary charge" is three times as large as the "quantum of charge". However all particles that can be isolated have charges that are integer multiples of *e*. Quarks only exist in collective states. In other words 'quantum of charge' may be ambiguous while 'elementary charge' is not.

17. A

When the droplet is suspended the free body diagram will be consistent with force equilibrium. The electrostatic force upwards is equal and opposite to the weight downwards.



The magnitudes of the two forces must be equal.

$$Eq = mg$$

The voltage between the plates is the product of the electric field and the distance of plate separation.

$$V = Ed$$

$$E = \frac{V}{d}$$

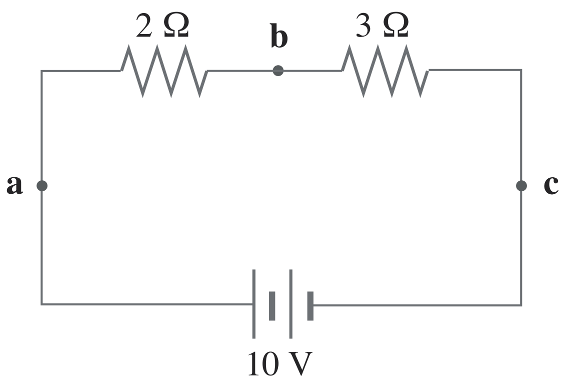
The force equilibrium expression allows us to express the voltage required as a function of droplet mass, charge and plate separation.

$$\frac{Vq}{d} = mg$$

$$V = \frac{mgd}{q}$$

DC Circuits Practice Items

The diagram below pertains to questions 1-6.



1. In what manner does current flow through the circuit?

- **A.** clockwise
- **B.** counter-clockwise
- **C.** there is no current
- **D.** alternating current

2. What is the value of the current at point **a**?

- **A.** 0 A
- **B.** 2.0 A
- **C.** 5.0 A
- **D.** 8.3 A

3. What is the value of the current at point **b**?

- **A.** 0 A
- **B.** 2.0 A
- **C.** 5.0 A
- **D.** 8.3 A

4. What is the potential difference between points **b** and **c**?

- **A.** – 6 V
- **B.** 2 V
- **C.** 6 V
- **D.** 10 V

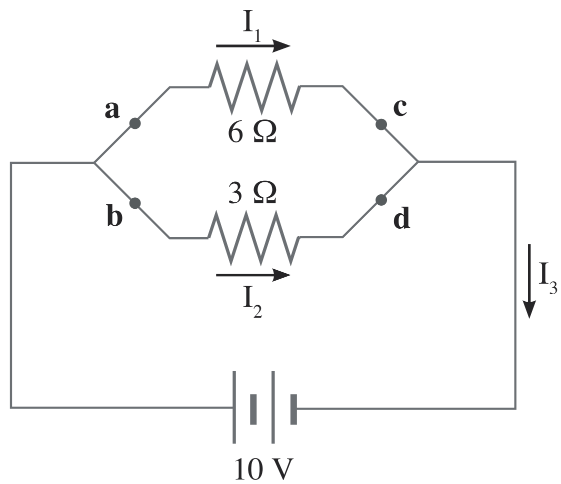
5. What is the potential difference between points **c** and **a**?

- **A.** – 10 V
- **B.** – 2 V
- **C.** 2 V
- **D.** 10 V

6. If the two resistors in the circuit are heating elements submerged in 0.5 liters of water, approximately how long would it take to raise the temperature 1 ^{o}C?

- **A.** 25 seconds
- **B.** 0.5 seconds
- **C.** almost two minutes
- **D.** about two hours

The diagram below pertains to questions 7 - 12.



7. In the circuit pictured above, what is the potential difference between points **b** and **c**?

A. – 10 V
B. – 3.3 V
C. 3.3 V
D. 10 V

8. What is the value of I_3?

A. 1.1 A
B. 3.3 A
C. 5 A
D. 20 A

9. What is the value of I_2?

A. 1.1 A
B. 3.3 A
C. 5 A
D. 20 A

10. What is the value of the power consumption of the entire circuit?

A. 2.0 W
B. 10 W
C. 20 W
D. 50 W

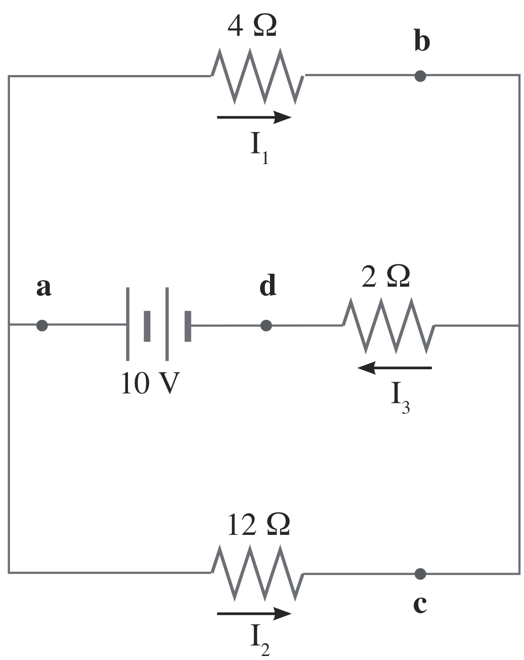
11. Assuming the power supply remained unchanged in delivering 10V to the external circuit, if the wire were cut at point b, which of the following would occur?

A. I_1 would increase
B. I_1 would decrease
C. I_2 would increase
D. I_3 would decrease

12. Which of the following statements is **untrue**?

A. I_2 is twice the value of I_1
B. $I_1 + I_2 = I_3$
C. The power consumption of the 6 Ω resistor is equal to the power consumption of the 3 Ω resistor.
D. all are true

The diagram below pertains to questions 13 - 15.



13. What is the value of I_3?

A. 0.63 A
B. 2.0 A
C. 3.3 A
D. 8.3 A

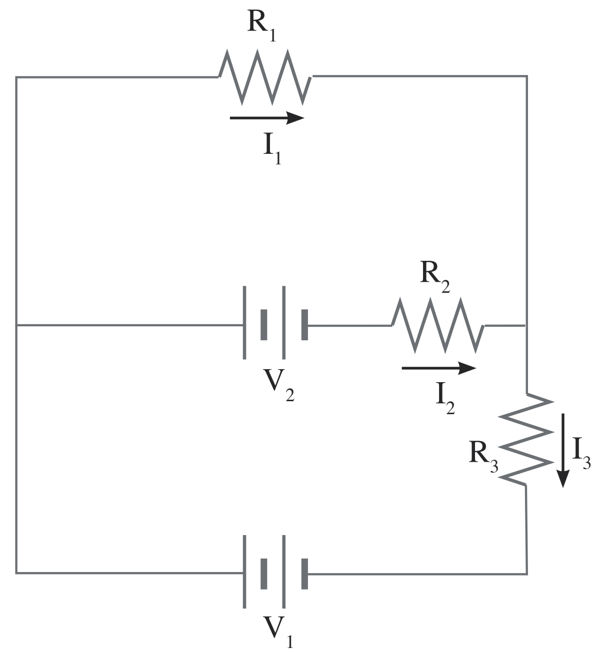
14. What is the value of the power consumed by the 12 Ω resistor?

A. 6 W
B. 3 W
C. 72 W
D. 25/3 W

15. Which of the following statements is **untrue**?

A. The potential difference between points b and c is zero.
B. I_1 is greater than I_2.
C. The potential difference between points b and d is 10 V.
D. All are true.

The diagram below pertains to questions 16 - 17.



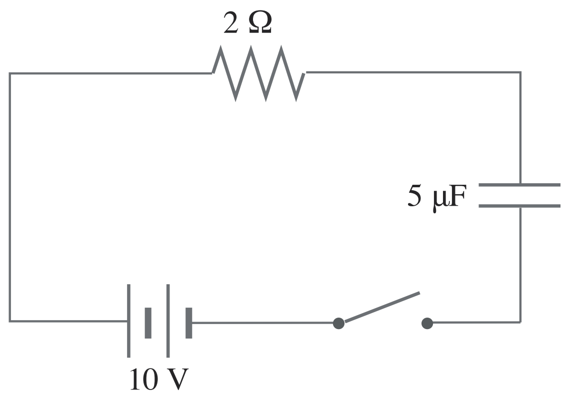
16. Which of the following expressions is incorrect?

A. $V_1 - I_1R_1 - I_3R_3 = 0$
B. $V_1 - V_2 - I_2R_2 - I_3R_3 = 0$
C. $I_1 + I_2 = I_3$
D. all are correct

17. If instead of the voltage source, V_2, a fully charged capacitor were present at that same position in the circuit, which of the following would occur?

A. I_1 would equal I_3.
B. The rest of the circuit would attain a net positive charge.
C. I_1 would be greater than I_3.
D. The voltage drop across the capacitor would be equal and opposite to V_1.

The diagram below pertains to questions 18 - 20.



18. In the RC circuit above, what is the maximum rate of power consumption by the resistor?

A. 5 W
B. 25 W
C. 50 W
D. 100 W

19. When the capacitor has been fully charged, how much charge will it hold?

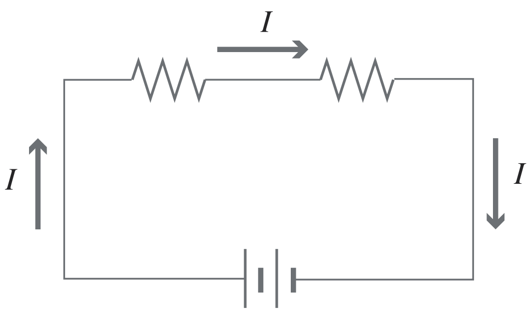
A. 50 µC
B. 50 mC
C. 2.0 C
D. 2.0×10^6 C

20. Which of the following statements predicts the behavior of the circuit?

A. Immediately after the switch has been closed, the potential drop will be entirely across the resistor.
B. After the capacitor has been fully charged, the potential drop will be entirely across the capacitor.
C. Employing a dielectric within the capacitor will increase the amount of charge stored in the fully charged capacitor.
D. all of the above

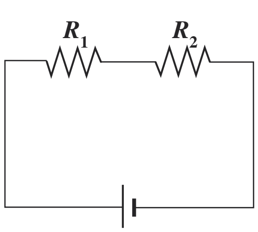
DC Current
Answers and Explanations

1. A
Electrical current is always represented as the net flow of positive charge, even if the charge carriers are electrons such as in a metallic conductor. The current flows from the positive terminal of the voltage source (long bar) to the negative terminal.



The positive terminal is the zone of positive potential in the circuit. This is where positive charges have high potential energy. It's good to visualize the charges 'falling' through the circuit from the positive to the negative terminal of the voltage source.

2. B
All of the external circuit elements are in series so the current will be the same throughout the circuit. To determine the current first we need to resolve the two series resistors as an equivalent resistance. Because we know the voltage across the two combined, we can then use Ohm's law to determine the current. The equivalent resistance of resistors in series is the sum of the individual resistances. This means that the equivalent resistance is greater than that of any individual resistor.



$$R_{ser} = R_1 + R_2 + R_3 + \cdots$$

First determine the equivalent resistance.

$$R_{ser} = 2\Omega + 3\Omega = 5\Omega$$

Now we can compute the current.

$$V = IR$$

$$I = \frac{V}{R}$$

$$I = \frac{10\text{ V}}{5\,\Omega} = 2.0\text{ A}$$

3. B
All of the external circuit elements are in series so the current will be the same at point **b** as at **a**.

4. A
The potential difference between points **b** and **c** is the change in potential energy per coulomb of charge moved from **b** to **c**. The charges in the current are losing potential energy as they cross the 3Ω resistor so the potential difference between the two points will be a negative value. We refer to this decrease in potential as the voltage drop across the 3Ω resistor.

To determine the voltage drop across the resistor, we just need to use Ohm's law applied to just that resistor.

$$V = IR = (2\text{A})(3\Omega) = 6\text{V}$$

5. D
The potential energy of a charge increases from **c** to **a** across the 10V voltage source so the potential difference is positive 10V.

6. C
First we need to ask how much energy is necessary to raise the temperature of 500g (0.5 L) of water one degree Celsius (1°C)? The specific heat of water is 1 cal/g°C so we're talking about 500 calories. 1 cal = 4.18J so this is approximately 2000J. (2092 J to be exact, but the question said we could be approximate, and the answers are widely spaced, so we're safe to make things easier for ourselves).

To find out how long this will take, we need to know the *power* expended in the resistors. Power is the rate at which energy is supplied to the resistors. Power is the product of the current and the voltage.

$$P = IV$$

2 coulombs per second (2A) of current falling through 10 joules per coulomb (10V) is consuming 20 J/s or 20 W of power.

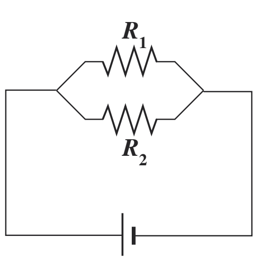
20W of power requires about 100s to deliver 2000J.

7. A

The voltage drop across parallel circuit elements is the same. The electrostatic force is a conservative force, so the change in potential energy between two points doesn't depend on the path between them. **a** is equipotential to **b** and they are both equipotential to the positive pole of the 10V voltage source. **c** is equipotential to **d** and they are both equipotential to the negative pole of the voltage source. Therefore the potential difference between points **b** and **c** is a 10V voltage drop.

8. C

To determine the primary current in the circuit first we need to resolve the two parallel resistors as an equivalent resistance. Because we know the voltage across the two, we can then use Ohm's law to determine the current. For parallel resistors, the reciprocal of the equivalent resistance equals the sum of the reciprocals of the individual resistances. The equivalent resistance in parallel is less than the resistance of any individual parallel resistor.



$$\frac{1}{R_{\text{par}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

First determine the equivalent resistance.

$$\frac{1}{R_{\text{par}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

When there are just two parallel resistors, there is an alternative way to express the formula above that makes the arithmetic a lot easier.

$$R_{\text{par}} = \frac{R_1 R_2}{R_1 + R_2}$$

$$R_{\text{par}} = \frac{(6\Omega)(3\Omega)}{(6\Omega + 3\Omega)} = 2\Omega$$

Now we can compute the current.

$$V = IR$$

$$I = \frac{V}{R}$$

$$I = \frac{10\text{ V}}{2\,\Omega} = 5.0\text{ A}$$

9. B

I_2 is the branch of the current flowing through the 3Ω resistor. The entire 10V acts across each branch, so we just need to use Ohm's law.

$$V = IR$$

$$I = \frac{V}{R}$$

$$I = \frac{10\text{ V}}{3\,\Omega} = 3.3\text{ A}$$

10. D

In question 8 we determined the value of the primary current in the circuit, I_3, to be 5A.

Power is the rate at which energy is supplied to the resistors. Power is the product of the current and the voltage.

$$P = IV$$

To determine the power,

$$(5\text{ A})(10\text{ V}) = 50\text{ W}$$

11. D

The equivalent resistance in parallel is less than the resistance of any individual parallel resistor. Cutting the wire at point **b** changes the circuit from one with a 2Ω external resistance (the equivalent resistance of the 3Ω and 6Ω parallel arrangement) into one with a single external 6Ω resistor. Increasing the external resistance will decrease the current, I_3, by Ohm's law.

$$V = IR$$

$$I = \frac{V}{R}$$

12. C

The same 10V is operating across the 3Ω and 6Ω resistors. Current follows the path of least resistance. We can see by Ohm's law that the current is has twice the value through the 3Ω compared to the 6Ω resistor.

$$V = IR$$

$$I = \frac{V}{R}$$

Power is the product of the current and the voltage.

$$P = IV$$

With the same drop in electrical potential across both resistors, the greater current flowing through the 3Ω resistor corresponds to a greater power output. The power consumption is less in the 6Ω resistor.

13. B

In this circuit a set of two parallel resistors (4Ω and 12Ω) are in series with a third resistor (2Ω). The straightforward way to solve this problem is to determine the equivalent resistance of the two parallel resistors first.

$$\frac{1}{R_{par}} = \frac{1}{R_1} + \frac{1}{R_2}$$

When there are just two parallel resistors, there is an alternative way to express the formula above that makes the arithmetic a lot easier.

$$R_{par} = \frac{R_1 R_2}{R_1 + R_2}$$

$$R_{par} = \frac{(12\Omega)(4\Omega)}{(12\Omega + 4\Omega)} = 3\Omega$$

So we can treat the parallel resistor as a single 3Ω resistor in series with the 2Ω resistor. The equivalent resistance of resistors in series is the sum of their individual resistances.

$$R_{ser} = R_1 + R_2 + R_3 + \cdots$$

So the total external resistance is 5Ω. Now we can compute the current, I_3.

$$V = IR$$

$$I = \frac{V}{R}$$

$$I = \frac{10\text{ V}}{5\,\Omega} = 2.0\text{ A}$$

14. B

In the previous problem we determined the equivalent resistance of the two parallel resistors (4Ω and 12Ω) to be 3Ω. These two as a set are in series with a third 2Ω resistor, so the total equivalent resistance of the external resistors is 5Ω. With a 10V voltage source, the primary current will be 2A, which we showed in the previous problem.

To determine the voltage drop across the 12Ω resistor, first remember that the voltage drop is the same across both resistors in parallel. As a set they have a 3Ω equivalent resistance. A primary current of 2A is passing through the set. Ohm's law (*V* = *IR*) tells us that 6V is required to drive 2A through 3Ω resistance. The voltage drop across the parallel resistors, and thus our 12Ω resistor, is 6V. 6V across a 12Ω resistor will operate at 3W power.

$$P = IV = I^2R = \frac{V^2}{R}$$

$$P = \frac{(6V)^2}{12\Omega} = 3W$$

15. C

Points **b** and **d** are separated by the 2Ω resistor. The entire 10V drop in potential between the positive and negative terminals of the voltage source cannot be occurring all across just that resistor because it is in series with the set of parallel resistors. The voltage falls part of the way as current expends energy in the parallel set of resistors first (4Ω and 12Ω) and then the voltage falls the rest of the way across the 2Ω resistor. We learned earlier that the current through the 2Ω resistor was 2A. Ohm's law (*V* = *IR*) tells us that 4V is required to drive 2A through 2Ω resistance. In other words, the potential difference between points **b** and **d** is 4V not 10V.

16. D

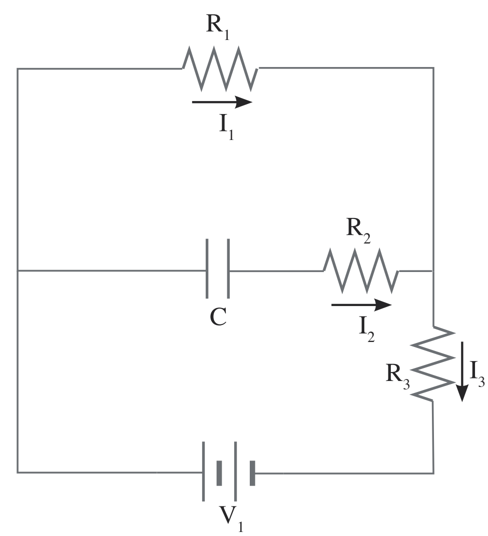
Statements **A** and **B** are instances demonstrating Kirchhoff's second rule: The sum of the changes in potential around any closed path is zero. Statement **C** derives from Kirchhoff's first rule: The sum of the currents into a junction equals the sum of currents out of the junction.

17. A

The figure below depicts the circuit with a fully charged capacitor substituted for V_2

With the capacitor fully charged, no current is flowing through the central branch and resistor R_2. In that case I_1 will equal I_3.

Regarding choice **B**, if this were a simple RC circuit, the voltage drop across a fully charged capacitor would e equal opposite to the voltage of the voltage source. In this circuit, however, the voltage drop of the capacitor plus the voltage drop across R_3 will be equal and opposite to the voltage source.



18. C

At the moment the capacitor is just beginning to be charged by the current, after the switch is closed, current will flow through the circuit as if the capacitor were not even there. As charge builds up on its plates, though, its voltage will begin to oppose the charging voltage and the current will begin to slow ultimately down to zero when the capacitor is fully charged and possesses a voltage equal and opposite to the charging voltage.

After the switch is closed, the entire voltage crop will be across the resistor in that moment, so the instantaneous power consumption by the resistor will be:

$$P = IV = I^2R = \frac{V^2}{R}$$

$$P = \frac{(10V)^2}{2\Omega} = 50W$$

19. A

Capacitance reflects the ability of a conductor to store charge. Capacitance is expressed as the relationship between the voltage of the conductor and the amount of charge stored. Capacitance is measured in farads (F). One farad is equal to one coulomb per volt.

$$C = \frac{Q}{V}$$

The charge stored will equal the product of the voltage across the capacitor and its capacitance.

$$Q = CV$$

$$Q = (5\ \mu\text{F})(10\text{V})$$

$$Q = (5 \times 10^{-6}\ \text{F})(10\text{V}) = 5 \times 10^{-5}\ \text{C}$$

$$= 50\ \mu\text{C}$$

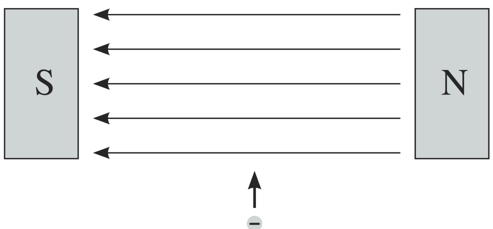
20. D

All three statements are correct. When the switch is closed, all of the voltage drop in the circuit is across the resistor, but eventually, as charge builds up on its plates, the voltage drop shifts to the capacitor. When the capacitor is fully charged, current in the circuit will have ceased. However, if a dielectric were between the plates of the capacitor, the capacitor could hold a larger amount of charge before obtaining the same charging voltage.

Magnetism Practice Items

1. What is the direction of the force on a negatively charged particle as it enters the magnetic field shown below?

A. to the left
B. to the right
C. into the page
D. out of the page



2. What is the magnitude and direction of a magnetic force acting on an electron (charge -1.6×10^{-19} C) moving in the opposite direction at 1000 m/s in the same plane and parallel to a 10 μT uniform magnetic field?

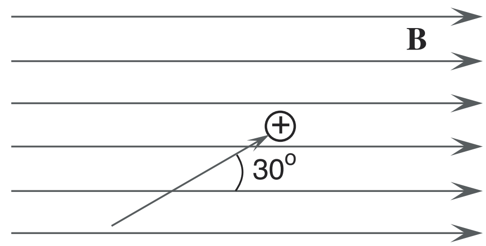
A. 0 N
B. 1.6×10^{-21} N out of the plane
C. 1.6×10^{-15} N into the plane
D. 1.6×10^{-15} N parallel to the plane

3. A positively charged particle is released from rest in a region where there is a uniform electric field and a uniform magnetic field. If the two fields are parallel to each other, the path of the particle is a

A. circle.
B. parabola.
C. helix.
D. straight line.

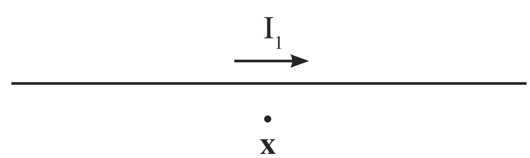
4. A proton (charge 1.6×10^{-19} C) moves at an angle to a the uniform magnetic field, **B** (300 Tesla) in the same plane as the field. The speed of the proton is 2×10^{6} m/s. What is magnetic force acting upon the proton?

A. 1.6×10^{-13} N, directed into the page
B. 4.8×10^{-11} N, directed into the page
C. 4.8×10^{-11} N, directed out of the page
D. 8.3×10^{-11} N, directed out of the page



5. What is the direction of the magnetic field at the point in space **x** near the current carrying wire pictured below?

A. Out of the page
B. Into the page
C. Upward in the plane of the page
D. Downward in the plane of the page

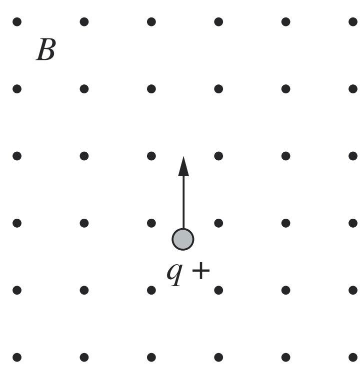


6. When electric current is flowing in the same direction through two, adjacent, parallel wires:

A. The wires attract each other.
B. The wires repel each other.
C. The wires exert no force on each others.
D. The wires oscillate.

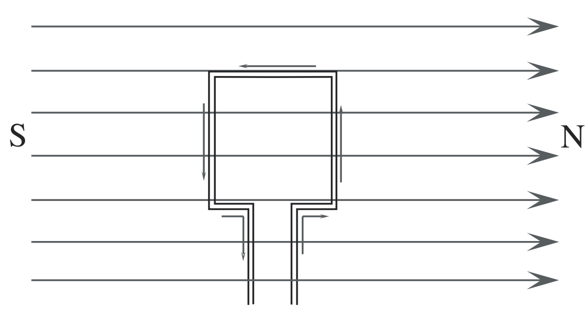
7. A positively charged particle is moving in the plane of the page in a direction perpendicular to the uniform magnetic field, ***B***, which points out of the page perpendicular to the plane of the page in the figure below. If the only force acting the particle is magnetic force, which of the following descriptions best applies to the motion of the particle?

A. clockwise circular motion
B. counter-clockwise circular motion
C. clockwise helical motion out of the paper
D. counter-clockwise helical motion into the paper



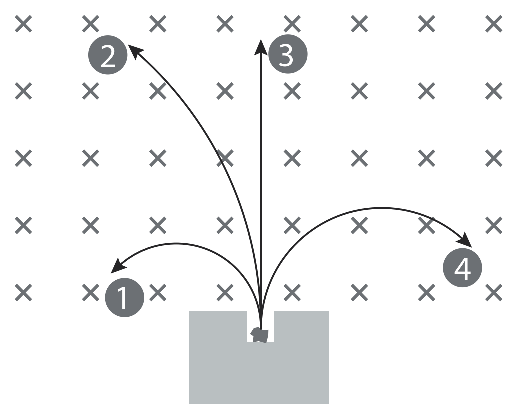
8. When current is flowing in the direction shown in the conducting loop below

A. the loop experiences a net torque.
B. the experiences a net force into the page.
C. the loop experiences compressive stress.
D. the loop is in a state of static equilibrium.



9. Radioactive emissions from an alloy in which several decay processes are occurring pass into a uniform magnetic field pointing into the page as shown in the figure below. The identity of particle 4 is most likely

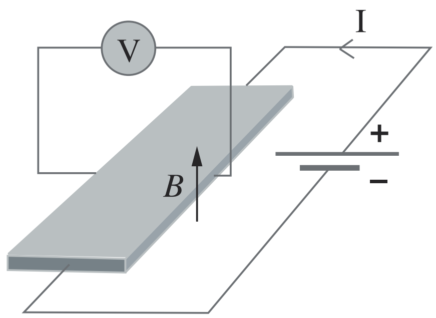
A. β^+
B. β^-
C. α
D. γ



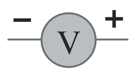
10. In a cyclotron particle accelerator, a beam of charged particles travels repeatedly round a loop. The purpose of the powerful magnets in a cyclotron is

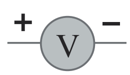
A. to record particle collisions.
B. to pull charged particles forward along the accelerator.
C. to steer and focus the particles.
D. to prevent the particles from colliding with gas molecules within the accelerator.

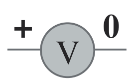
11. A voltage source is connected in series by copper wires to a steel plate, which is oriented perpendicular to the magnetic field, ***B***, as shown in the figure below.

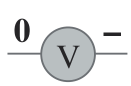


While current is flowing through the circuit, which of the following best represents the potential difference at the voltmeter pictured in the diagram above?

A. 

B. 

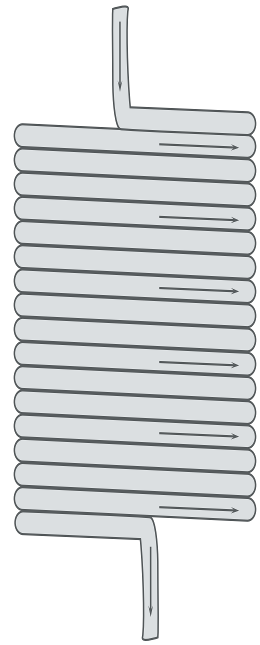
C. 

D. 

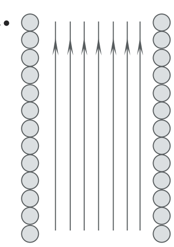
12. When subjected to an applied magnetic field, a material is magnetized in the same direction as the applied magnetic field with the induced field directly proportional to the applied field strength. This material is

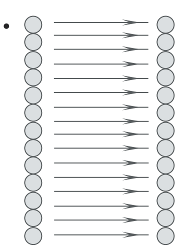
A. diamagnetic
B. paramagnetic
C. ferromagnetic
D. ferrimagnetic

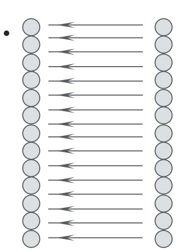
13. A coil of conducting wire is wound into a tightly packed helix. An electric current passing through the wire travels from left to right across the near face of the coil as shown in the figure below:

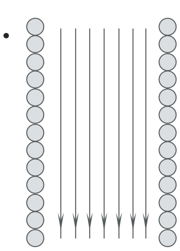


Which of the following most closely depicts the magnetic field in the volume of space within the coil?

A. 

B. 

C. 

D. 

The following passage pertains to questions 14 - 19.

Assigning oxygenated hemoglobin's oxidation state is difficult because oxyhemoglobin (Hb-O_2), by experimental measurement, is diamagnetic, yet the low-energy electron configurations in both oxygen and iron are paramagnetic.

Triplet oxygen, O_2, the lowest-energy molecular oxygen species, has two unpaired electrons in anti-bonding π* molecular orbitals. Iron(II) tends to exist in a high-spin configuration with unpaired electrons spread out among its d orbitals in accord with Hund's rule. Iron(III) has an odd number of electrons, and thus must have one or more unpaired electrons, in any energy state. Thus, a non-intuitive distribution of electrons in the combination of iron and oxygen must exist, in order to explain the observed diamagnetism and no unpaired electrons.

The three logical possibilities to produce diamagnetic (no net spin) Hb-O_2 are as follows. One possibility is that low-spin Fe^{2+} binds to singlet oxygen. Both low-spin iron and singlet oxygen are diamagnetic. However, the singlet form of oxygen is the higher-energy form of the molecule. Another possibility is for low-spin Fe^{3+} to be bound to O^{2-} (the superoxide ion) and the two unpaired electrons couple antiferromagnetically, giving diamagnetic properties. The third possibility is that low-spin Fe^{4+} binds to peroxide, O_2^{2-}. Both are diamagnetic.

Compelling evidence from X-ray photoelectron spectroscopy and IR spectroscopy point to the second choice as the one that is correct of the three logical possibilities for diamagnetic oxyhemoglobin. This is not surprising because singlet oxygen (possibility #1) and large separations of charge (possibility #3) are both unfavorably high-energy states. Iron's shift to a higher oxidation state in Hb-O_2 decreases the atom's size, and allows it into the plane of the porphyrin ring, pulling on the coordinated histidine residue and initiating the allosteric changes seen in the globulins. All three models for diamagnetic Hb-O_2 may contribute to some small degree (by resonance) to the actual electronic configuration of Hb-O_2. However, the model of iron in Hb-O_2 being Fe(III) is more correct than the classical idea that it remains Fe(II).

14. As described in the passage which trait do triplet oxygen, high spin iron II, and iron III all have in common?

A. They are paramagnetic.
B. They have an even number of electrons.
C. They contribute to the structure of oxyhemoglobin by resonance.
D. They represent unfavorably high energy states.

15. Which medical imaging technique might be used to detect the relative abundance of the oxygenated and deoxygenated forms of hemoglobin in living tissue?

A. functional magnetic resonance imaging (fMRI)
B. electroencephalography (EEG)
C. positron emission tomography (PET)
D. computed tomography (CT)

16. Ferritin is a ubiquitous intracellular protein that stores iron and releases it in a controlled fashion. Based on information presented in the passage, ferritin is most likely

A. diamagnetic.
B. paramagnetic.
C. ferromagnetic.
D. anti-ferromagnetic.

17. According to the information presented in the passage, which value for bond order would most likely correspond to the O-O bond in Hb-O_2?

A. 1
B. 1.5
C. 2
D. 3

18. Which of the following information in the passage supports the determination that low-spin iron(III) bound to superoxide represents the iron-oxygen binding state in Hb-O_2.

A. Both Fe^{3+} and O^{2-} are diamagnetic.
B. Spectroscopic evidence that Hb-O_2 is paramagnetic.
C. Infrared vibrational frequencies of the O-O bond suggests a bond length consistent with singlet oxygen.
D. Fe^{3+} has a smaller radius than Fe^{2+}.

19. Which of the following is suggested in the passage to explain the diamagnetic properties of Hb-O_2 given that neither iron(III) and superoxide are diamagnetic?

A. Neighboring singlet electrons may align with spins pointing in opposite directions.
B. Electron sharing between O^{2-} and high-spin Fe^{3+} produces a hybrid orbital.
C. A decrease in the size of the iron atoms allows it into the plane of the porphyrin ring.
D. Binding with superoxide forces the iron(III) *d* orbitals into a spherical arrangement in which they are degenerate.

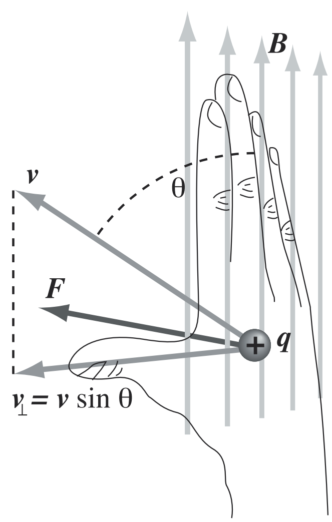
Magnetism

Answers and Explanations

1. C

A magnetic force is produced on a particle proportional to the charge, field strength and the component of its velocity perpendicular to a magnetic field. The force produced is perpendicular to both the particle velocity and to the field.

We have a right hand rule to determine the direction of the magnetic force on a positively charged particle. Align your thumb in the direction of the component of particle velocity perpendicular to the field. Align your fingers in the direction of the magnetic field. The direction of the magnetic force vector is pointing out of your palm.



If the charge moving into the magnetic field in the problem were a positive charge, the resulting magnetic force on it as it enters the field would be out of the page. However, because it is a negative charge, the orientation of the force is reversed. The particle experiences a force into the page.

2. A

The magnetic force acting on a particle moving through a magnetic field is proportional to the component of the particle's velocity perpendicular to the field. The particle's velocity in this problem, however, is completely parallel to the magnetic field, so no magnetic force results.

3. D

The charged particle will experience an electrostatic force in the direction of the electric field and begin to accelerate in that direction. Because the particle's velocity will be completely parallel to the magnetic field, no magnetic force results, so it will continue moving in a straight line.

4. D

The magnitude of the magnetic force is the product of the charge, the speed of the particle, the magnetic field strength and the sine of the angle between particle velocity and the field. (You can also think of it as the product of the charge, the field, and the speed perpendicular to the field).

$$F = qvB \sin\theta$$

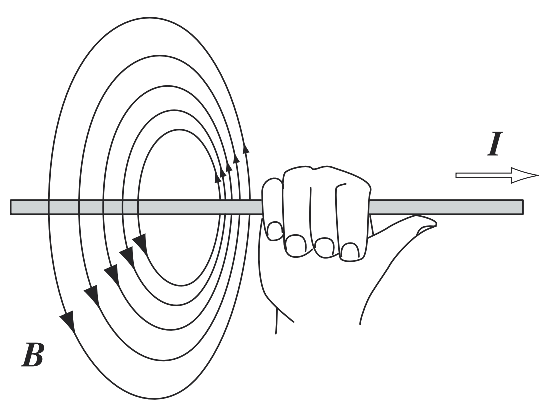
$$= (1.6 \text{ X } 10^{-19}\text{ C})\ (2.0 \text{ X } 10^{6}\text{ m/s})\ (300\text{ T})\ (0.5)$$

$$= 4.8 \text{ X } 10^{-11}\text{ N}$$

Orienting our right hand rule with our thumb in the direction of the velocity component perpendicular to the field predicts a force into the page for a positively charged particle into the page.

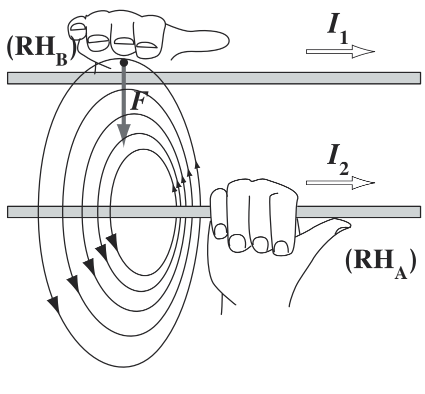
5. B

The magnetic field of a straight current carrying wire encircles the wire. With the thumb of your right hand pointed in the direction of positive current, wrapping your fingers around the wire gives the orientation of the magnetic field.



6. A

Use both right hand rules: Use the first rule, (**RH$_A$**), to predict the orientation of a field produced by the current of one of the wires. The second rule, **(RH$_B$)**, predicts the orientation of the magnetic force. In the figure below, we see that the magnetic field produced by current I_2 exerts an attractive force on current I_1. The two wires attract each other.

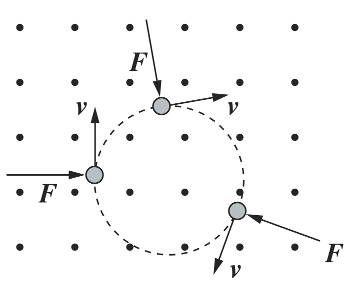


RH$_A$ – Point the thumb of your right hand in the direction of positive current, then wrap your fingers around the wire to show orientation of the magnetic field.

RH$_B$ – With your thumb of your right hand in the direction of the component of the current that is perpendicular to the magnetic field from the other wire and your fingers in the direction of that field, the direction of the magnetic force is out of your palm.

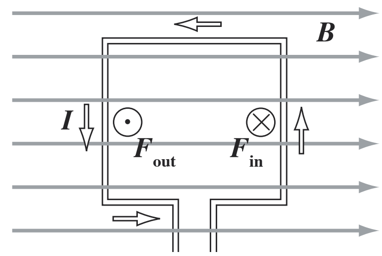
7. B

The magnetic force is perpendicular both to the particle velocity and to the magnetic field. The particle moves in a circle. The magnetic force is a centripetal force.



8. A

If the plane of a horizontal current loop is parallel to a magnetic field, a downward magnetic force is produced on one side of the loop and an upward force on the other. The result is a net torque on the loop that increases with the current, loop area, and magnetic field strength.



9. A

Orienting our right hand with our fingers straight in the direction of the field lines into the page and our thumb in the direction of the velocity component perpendicular to the field predicts a force to the left for a positively charged particle. Therefore, particle 4 must necessarily be negatively charged.

The only negatively charged particle of the choices is β^-**.**

10. C

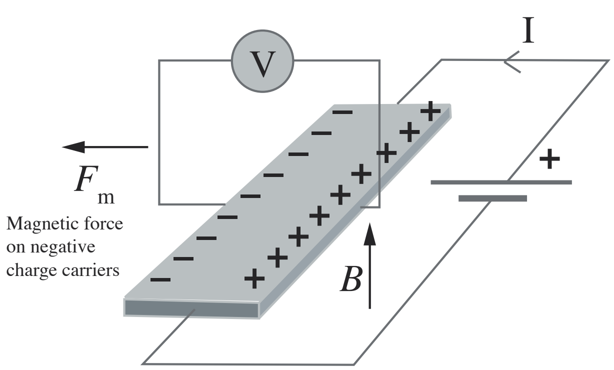
A charged particle moving perpendicular to a static uniform magnetic field will move in a circle due to magnetic force. The circular motion may be superimposed with an axial motion, resulting in a helix. In other words, charged particles spiral around magnetic field lines. This is how the magnets within a cyclotron can be made to steer and focus the particle beam.

The key to getting a question like this correct is not to be intimidated by out of scope reference. We can rule out choice '**B**' because the magnetic force is always perpendicular to the instantaneous velocity of the particle it acts on, so it does not perform work on the particle. The magnetic force steers the particles. It does not cause them to gain kinetic energy.

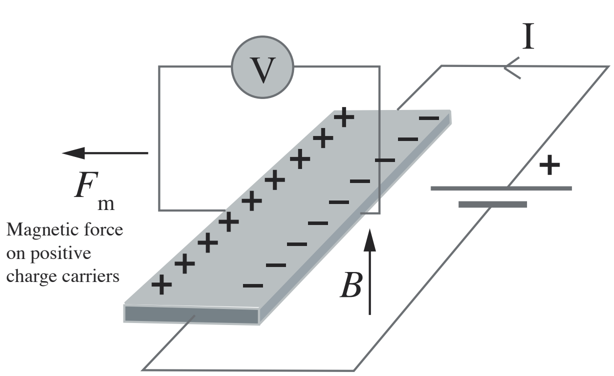
11. A

The magnetic field exerts a transverse force on the charge carriers moving through the flat conductor which tends to push them to one side of the conductor. The build-up of charge produces a measurable voltage between the two sides of the conductor. The presence of this measurable transverse voltage is called the Hall effect.

By convention, we always refer to the current as the flow of positive charge even when the charge carriers are electrons moving in the opposite direction, as you have in metallic conductors. However, the Hall effect is one of the few problems where the identity of the charge carriers makes a difference in the answer. For our problem involving a flat copper plate, the charge carriers are electrons flowing in the opposite direction of the positive current depicted in the figure. This produces the voltage as shown below:



You would have gotten a different answer if the current were actually composed of positive charge carriers flowing in the same direction as the current.



12. B

Paramagnetic materials possess unpaired electrons at the orbital level. Paramagnetic materials exhibit magnetization proportional to the strength of the external magnetic field in which the material is placed, and when the field is removed, the magnetization disappears.

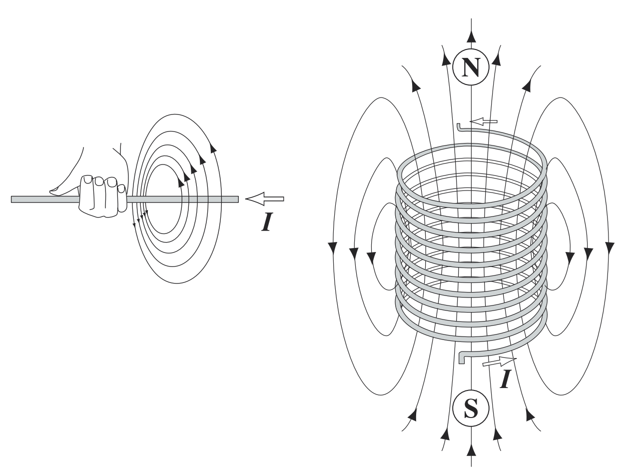
All electrons are paired in diamagnetic materials. When an external magnetic field is applied to a diamagnetic material, the tiny electron current loops at the atomic level align in such a way as to oppose the applied field. This magnetization also disappears after the external field is removed.

In ferromagnetic materials, unpaired electron spins line up parallel with each other in large scale magnetic domains. The bulk material is usually unmagnetized because the domains will be randomly oriented. However, in an external field the magnetic domains align and the ferromagnetic material becomes magnetized. Magnetization is rapid and nonlinear up to a saturation point. With ferromagnetic materials, there may also be remanence, meaning that magnetization may persist even after removal of the external field.

13. B

A current carrying conductor wound into a tight helix is known as a solenoid. The magnetic field inside a long, narrow, tightly wound solenoid is uniform.

Use the right hand for predicting the orientation of the magnetic field of a current to determine the orientation of the field within the coil:



14. A

Triplet oxygen, high spin iron II, and iron III all have unpaired electrons. As such, they are all three paramagnetic.

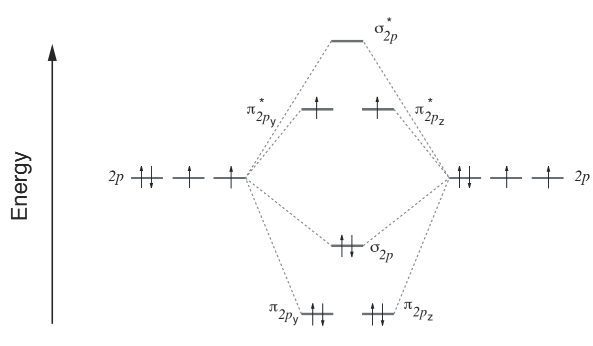
15. A

The basic underlying phenomenon making fMRI possible is that when oxyhemoglobin loses oxygen to become deoxyhemoglobin, it shifts from being diamagnetic to paramagnetic. In other words, the magnetic properties of blood are a function of oxygen saturation.

17. B

According to the passage, multiple lines of evidence support the hypothesis that the configuration of O_2 in oxyhemoglobin is superoxide radical anion.

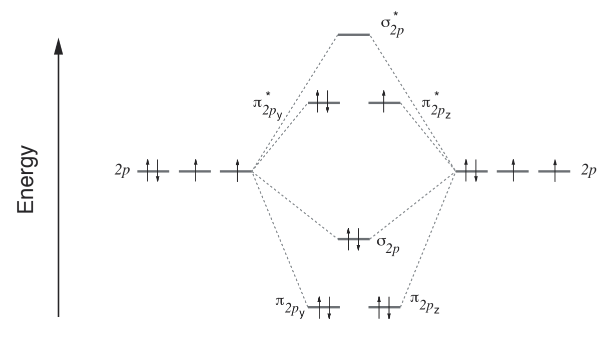
To understand the bond order in superoxide, it's very helpful if you have a mental picture of the molecular orbital diagram of normal ground state O_2 (sometimes called triplet oxygen after the number of its spin states). It's not too unreasonable to expect this mental picture to be present because O_2 is the classic example of a molecule whose Lewis structure makes it look diamagnetic but which is actually paramagnetic.



Ground state O_2

Just like the Lewis structure would show, the bond order of O_2 is 2, but after overlap of its *p* subshells the molecule yields three pairs of electrons in bonding orbitals and two singlets in anti-bonding orbitals. This is why the passage refers to O_2 as paramagnetic.

Superoxide radical anion would have the following molecular orbital diagram.



Superoxide O_2^-

There are six electrons in bonding orbitals and three in anti-bonding orbitals, so the bond order is 1.5.

18. D

The passage mentions that iron's shift to a higher oxidation state in Hb-O_2, which would be Fe^{3+}, decreases the atom's size, and allows it into the plane of the porphyrin ring. (Porphyrin is the organic component of the heme prosthetic group in hemoglobin).

19. A

Even though superoxide and low spin Iron III are both paramagnetic, Hb-O_2 is diamagnetic. That is a central theme in the second half of the passage. The plausible explanation given is antiferromagnetic coupling. Even if you have never encountered this concept, it should be clear that answer choice '**A**' is the only plausible description of antiferromagnetic coupling.

Properties of Light Practice Items

1. Which of the following types of electromagnetic radiation has the longest wavelength?

A. infrared radiation
B. gamma rays
C. microwaves
D. X-rays

2. The quantum nature of light is most important to which of the following phenomena?

I. Polarization
II. Interference
III. Photoelectric effect

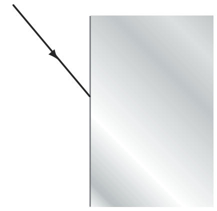
A. only I
B. only III
C. I and II
D. I, II, and III

3. Which of the following distinguishes electromagnetic waves from sound waves?

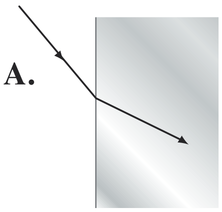
I. Electromagnetic waves can be polarized.
II. Elecromagnetic waves are transverse.
III. Electromagnetic waves carry energy.

A. only I
B. only II
C. I and II
D. I, II, and III

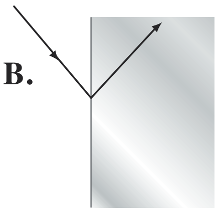
4. A light ray in air is incident on a glass block as shown at right. Which of the following diagrams represents the transmitted ray?



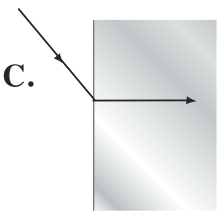
A.



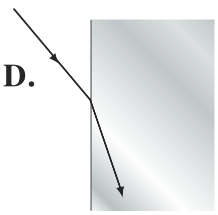
B.



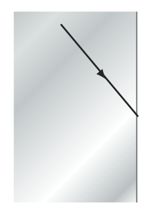
C.



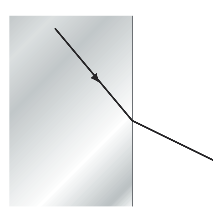
D.



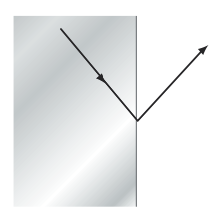
5. The incident angle for a light ray passing through the boundary from glass into air is less than the critical angle. Which of the following diagrams represents the transmitted ray?



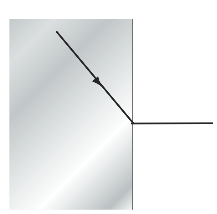
A.



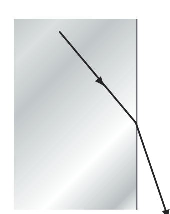
B.



C.



D.



6. A Crookes tube emits X-rays having a wavelength of 1.5Å in air. What is their frequency?

A. 2.0×10^{2} s^{-1}
B. 4.5×10^{8} s^{-1}
C. 5.0×10^{17} s^{-1}
D. 2.0×10^{18} s^{-1}

7. The wave number of a source of electromagnetic radiation is 1700 cm^{-1}. The type of electromagnetic radiation is

A. infrared
B. visible light
C. ultraviolet light
D. x-ray

8. The speed of light in glass (refractive index *n* = 1.5) is approximately

A. 1.5×10^{8} m/s
B. 2.0×10^{8} m/s
C. 3.0×10^{8} m/s
D. 4.5×10^{8} m/s

9. As visible light moves from air into water its wavelength

A. shortens
B. lengthens
C. remains the same
D. the answer depends on the angle of incidence

10. Which of the following best explains why gem cut diamonds sparkle?

A. high refractive index
B. high reflectance
C. low emittance
D. high transparency

11. When unpolarized light is incident on a transparent dielectric surface at Brewster's angle, the refracted light is

A. polarized
B. partially polarized
C. unpolarized
D. nonexistent

12. The transition energies associated with intramolecular vibration and rotation generally correspond to frequencies of

A. microwave radiation
B. infrared radiation
C. ultraviolet light
D. X-rays

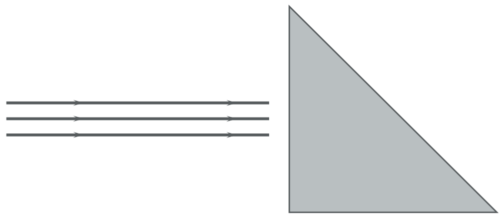
13. Light traveling through air is incident at an angle of 90°on a crown glass surface. As the light is transmitted into the glass its

A. frequency decreases
B. direction changes
C. wavelength decreases
D. frequency increases

14. When plane polarized light is passed through a circularly birefringent, or in other words, optically active fluid, the light will exit

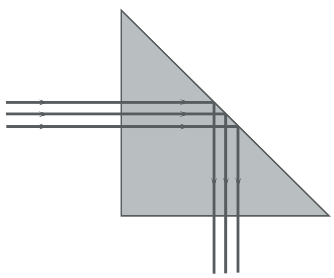
A. still linearly polarized but with the axis of polarization rotated.
B. with electric and magnetic fields rotating at the optical frequency.
C. exhibiting elliptical polarization.
D. split into two beams polarized in mutually orthogonal planes.

15. Suppose blue light were beamed perpendicular to the side of an isosceles right-triangular crown glass prism as shown below.

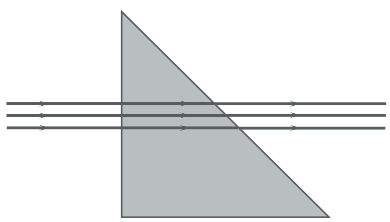


If the index of refraction of the prism is 1.52, which of the diagrams below correctly illustrates the result? (assume $n_{air} = 1$)

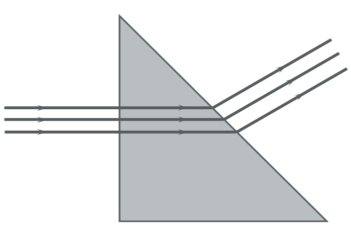
A.



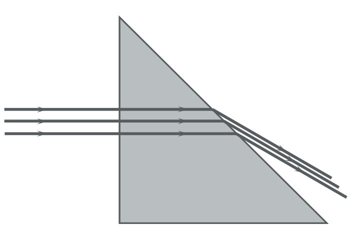
B.



C.



D.

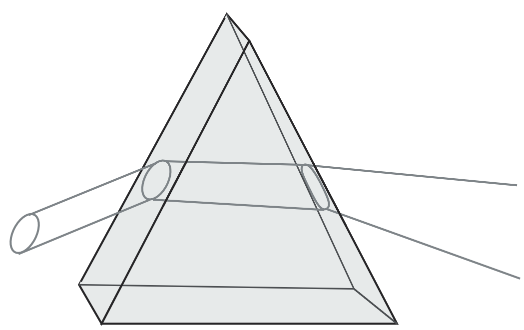


The following passage pertains to questions 16 - 19.

When light passes at an angle of incidence from one medium into another, refraction will occur because the speed of light is different within the two media. The ratio of the speed of light in a vacuum to the speed of light in a particular medium is called the index of refraction for that medium. The relationship between the angle of incidence, the angle of refraction, and the indices of refraction for the two media is given by Snell's law:

$$n_1 \sin \theta_i = n_2 \sin \theta_r$$

The index of refraction of a certain media is not constant for the entire electromagnetic spectrum, however, but depends somewhat upon the frequency. Most substances show increasing refractive index with increasing frequency of light. Such phenomena as dispersion of visible light into its spectral components by a prism, illustrated below, can be understood in terms of the variation of refractive index with frequency.



16. A prism separates white light into separate colors because

A. constructive interference occurs at different angles for different wavelengths.
B. the different path lengths through the glass lead to destructive interference for certain wavelengths.
C. white light is not coherent.
D. different colors of light have different indices of refraction in glass.

17. The index of refraction of crown glass is 1.53 for violet light. What will the approximate angle of refraction in glass equal if the angle of incidence at the air-glass boundary is 45˚?

A. 15˚
B. 30˚
C. 45˚
D. 60˚

18. According to the passage which of the following statements follows from the relationship between the index of refraction for glass and the properties of the incident light?

A. High frequency light is refracted with a smaller angle of refraction than low frequency light.
B. Orange light has longer wavelength than violet.
C. A low energy photon travels more slowly through glass than a high energy photon.
D. The refractive index is the ratio of the speed of light in vacuum and the phase speed of light in a material.

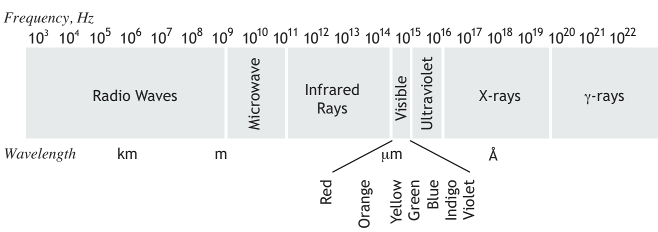
19. Orange light has a wavelength in air of 600 nm. What is its frequency?

A. 180 GHz
B. 500 GHz
C. 330 THz
D. 500 THz

Properties of Light
Answers and Explanations

1. C

There are not precise boundaries between the types of electromagnetic radiation. Radio waves are the longest wavelength type with wavelengths greater than approximately 1 meter ($f <$ 100MHz). With higher frequency and shorter wavelengths, microwaves are sometimes referred to as short-wavelength radio waves (30cm > λ > 1mm). Above microwaves on the spectrum, infrared radiation consists of wavelengths between 1mm up to the longest wavelengths of visible light (about 0.7μm). Visible light represents the narrow band in the spectrum (0.7μm > λ > 0.4μm) that the human eye can detect. Above visible light, with ever shorter wavelength and higher frequency, are ultraviolet light, x-rays, and gamma rays.



2. B

Both interference of light and polarization can be explained using a wave approach to electromagnetic radiation, the photoelectric effect is only explicable through quantum theory. In the photoelectric effect, electrons are dislodged from a metal surface only by the impingement of light that has reached or exceeded a threshold frequency. Below that threshold, no electrons are emitted from the material, regardless of the light intensity or the length of time of exposure to the light. To explain these results Einstein proposed that a beam of light is not a wave propagating through space, but rather a collection of discrete wave packets (photons).

3. C

Electromagnetic waves are transverse. Sound waves are longitudinal. Being transverse, electromagnetic waves may be polarized. In a transverse wave, the direction of the oscillation is perpendicular to the direction of motion of the wave. With a transverse wave, a principle of selection may apply with regard to the geometric orientation of the oscillation. In other words, transverse waves may be polarized, longitudinal waves may not.

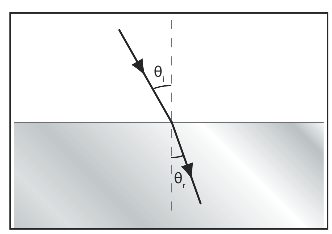
With regard to choice III, both sound waves and electromagnetic waves carry energy, so that property does not distinguish the two types of waves.

4. A

When a light ray travels between media, as it enters the new substance, the light ray is bent or refracted. According to Snell's Law, the product of the index of refraction in the first medium and the sine of the angle of incidence equals the product of the index of refraction in the second medium and the sine of the angle of refraction.

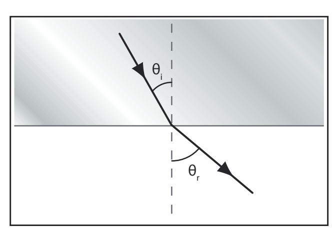
$$n_1 \sin\theta_i = n_2 \sin\theta_r$$

Consequently, light moving from a lower index of refraction medium to a higher index of refraction medium will bend towards the normal.



5. D

Light moving from a higher index of refraction medium to a lower index of refraction medium will bend away from the normal.



6. D

Because the speed of a wave is the product of the wavelength and frequency, you can divide either wavelength or frequency into the wave speed to get the other. The speed of light in air is very close to the speed of light in a vacuum, so we can use **c** here.

$$\lambda = 1.5\ \text{Å} = 1.5 \times 10^{-10}\ \text{m}$$

$$f = \frac{c}{\lambda}$$

$$f = \frac{3 \times 10^{8}\ \text{m/s}}{1.5 \times 10^{-10}\ \text{m}} = 2 \times 10^{18}\ \text{s}^{-1}$$

7. A

Using the dimensionless quantity 'cycle' we say that the wavelength of a harmonic wave tells us the 'meters per cycle', how much spatial distance is involved in a complete cycle. The wave number is the reciprocal of the wavelength. In other words, wave number tells you the number of waves per unit distance, the number of 'cycles per meter.'

The question asks us to identify the type of electromagnetic wave which has a wave number of 1700 cm^{-1}. To determine this corresponds to the infrared spectrum one strategy would be to take the reciprocal of 1700 cm^{-1}, convert from cm to nm, and we would determine a wavelength of approximately 6000 nm. This is longer than visible light, which has a wavelength up to 700 nm. Infrared is the only choice. (Infrared radiation extends from the nominal red edge of the visible spectrum at 700 nanometers (nm) to 1 millimeter (mm)).

However, it is much easier to answer this question quickly if you recognize that this value, 1700 cm^{-1}, is right in the middle of the abscissa of the typical IR spectrograph. In infrared (IR) spectrography the range of incident IR frequencies is designated using wave numbers demarcated using cm^{-1}. A large peak at 1700 cm^{-1} represents the characteristic IR absorption of the stretching vibration of the carbonyl group.

8. B

The index of refraction of a medium is defined as the ratio of the speed of light in a vacuum to the speed of light in the medium.

$$n = \frac{c}{v}$$

To determine the speed of light in a particular medium, divide the speed of light by the index of refraction.

$$v = \frac{c}{n}$$

$$v = \frac{3.0 \times 10^{8}\ \text{m/s}}{1.5}$$

$$v = 2.0 \times 10^{8}\ \text{m/s}$$

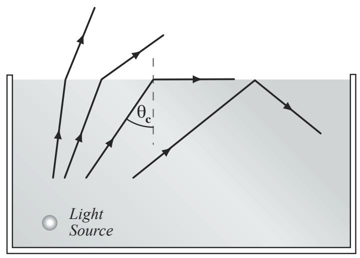
9. A

The speed of light within a medium is inversely proportional to its index of refraction. The greater the index of refraction the slower the medium. Because the frequency of a light ray is constant between media, it follows that the wavelength of the light must change. If speed slows but frequency is the same, the wavelength must now be shorter. In other words, wavelength and speed are directly proportional between media. Wavelength and index of refraction are inversely proportional.

$$\frac{\lambda_2}{\lambda_1} = \frac{v_2}{v_1} \qquad \frac{\lambda_2}{\lambda_1} = \frac{n_1}{n_2}$$

10. A

Light moving at an angle from a higher index of refraction medium to a lower index of refraction medium bends away from the normal. According to Snell's Law, in such cases where $n_1 > n_2$, there is an angle of incidence that corresponds to a 90° angle of refraction. From this angle of incidence, the refracted ray is predicted to move parallel to the interface between the two media. This angle is called the critical angle, θ_c. Light impinging at an angle of incidence greater than the critical angle will not be refracted but internally reflected at the interface. (The sine of the critical angle is simply the ratio n_2/n_1 because $\sin 90° = 1$).



$$\sin\theta_c = \frac{n_2}{n_1}$$

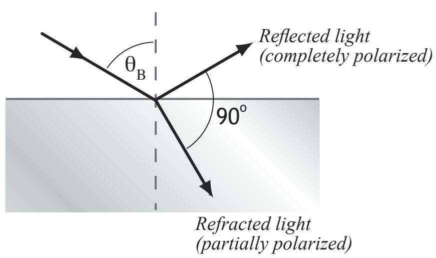
$$(n_1 > n_2)$$

Most of the light entering the front facet of a gem cut diamond undergoes internal reflection off the diamond-air boundary of the back facet. Diamond has a very high refractive index, $n = 2.4$. Because its index of refraction is so high, the critical angle (above which internal reflection occurs) is very low.

11. B

When light is incident at Brewster's angle the *reflected* light will be completely polarized.

$$\tan\theta_B = \frac{n_2}{n_1}$$



The *refracted* light will be partially polarized.

12. B

Infrared radiation is emitted or absorbed by molecules when they change their rotational-vibrational movements. The various stretching, scissoring, wagging, and twisting frequencies associated with bonded atoms vibrating as quantum oscillators within molecules overlap with the frequencies of infrared radiation. Infrared spectroscopy examines absorption of infrared radiation as a method for chemical analysis.

13. C

Refraction occurs as the light enters the crown glass, but given that the light is incident perpendicular to the glass, there is no alteration in the direction of the light ray. The angle of incidence is zero, and the angle of refraction will also be zero. However, the speed of the light will decrease. Because the frequency of a light ray is constant between media, it follows that the wavelength of the light must decrease.

14. A

If plane polarized light passes through a chiral media, for example a solution of the pure isomeric form of a chiral molecule, the plane of polarization of the incident light will be rotated. This is called circular birefringence. This is a major topic within stereochemistry for both organic compounds and coordination complexes.

15. D

Light striking the glass-air interface at a 45° angle is internally reflected. 45° is greater than the critical angle for a crown glass-air interface. Remember that light moving from a high index of refraction medium to a low index medium bends away from the normal. However, light can only bend 90° before it's traveling along the interface, and the critical angle at which this occurs, and beyond which reflection occurs, will be lower, the greater the difference between the indices of refraction of the two media.

$$\sin\theta_c = \frac{n_2}{n_1} \qquad (n_1 > n_2)$$

To use mental math for this problem, remember that the sine of 45° is 0.71, while the n_2/n_1 in the problem would be 1.0/1.5 or 0.67, so 45° is above the critical angle.

16. D

As the passage describes, dispersion of visible light by a prism occurs because the index of refraction varies somewhat with the frequency of the light. The color of visible light is determined by its frequency.

17. B

According to Snell's Law, the product of the index of refraction in the first medium and the sine of the angle of incidence equals the product of the index of refraction in the second medium and the sine of the angle of refraction. Moving from a lower to a higher index of refraction causes the ray to bend towards the normal.

$$n_1 \sin \theta_i = n_2 \sin \theta_r$$

$$(1.0)(\sin 45°) = (1.5)(\sin \theta_r)$$

$$\sin \theta_r = \frac{(1.0)(0.71)}{(1.5)}$$

$$\sin \theta_r \sim 0.5$$

$$\theta_r \sim 30°$$

18. A

The passage describes how in glass there is increasing refractive index with increasing frequency of light. According to Snell's Law, the greater the index of refraction of the new medium, the smaller the angle of refraction will be. This means that higher frequency light in glass will bend slightly more towards the normal (the line perpendicular to the interface) when moving from air to glass.

Note that both choices **B** and **D** are true statements, but neither answers the question.

19. D

The first thing we should do is convert our wavelength into SI units.

$$600 \text{ nm} = 6.0 \times 10^{-7} \text{ m}$$

The speed of a wave is the product of the wavelength and frequency. You divide wavelength into the wave speed to get the frequency. The speed of light in air is very close to the speed of light in a vacuum, so we can use **c** here.

$$\lambda = 6.0 \times 10^{-7} \text{ m}$$

$$f = \frac{c}{\lambda}$$

$$f = \frac{3 \times 10^8 \text{ m/s}}{6.0 \times 10^{-7} \text{ m}} = 5.0 \times 10^{14} \text{ s}^{-1} = 500 \text{ THz}$$

Geometric Optics Practice Items

1. When an object is placed before a plane mirror the image created is

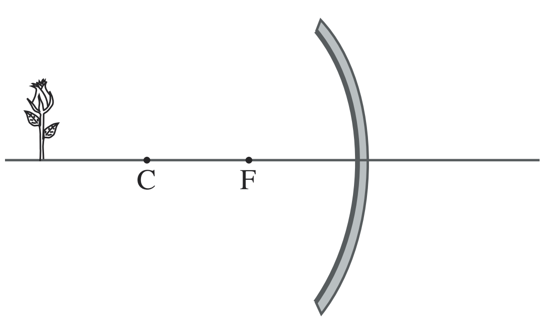
A. inverted, real, diminished
B. upright, real, the same size
C. inverted, virtual, the same size
D. upright, virtual, the same size

2. Which of these in a single lens or mirror system could produce a virtual image with a magnification of 0.7?

A. convex mirror
B. concave mirror
C. converging lens
D. convex lens

3. A flower is placed before a concave mirror as shown in the figure below. The image produced by the mirror is

A. virtual, upright, diminished
B. real, upright, enlarged
C. real, inverted, diminished
D. virtual, inverted, enlarged

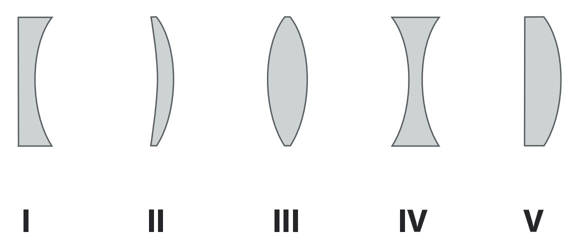


4. An object is placed at the center of the curvature, C, in front of a concave mirror with focal length, F. The image is located

A. the distance I = C
B. the distance I < F
C. the distance I = F
D. no image is formed

5. Which of the lenses shown in the figure above are converging lenses?

A. **I** and **IV**
B. **II**, **III** and **V**
C. **I** only
D. **III** and **V**

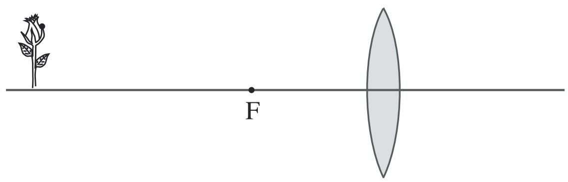


6. Referring to the same figure above as in the preceding question, which of the lenses are diverging lenses?

A. **I** and **IV**
B. **II**, **III** and **V**
C. **I** only
D. **III** and **V**

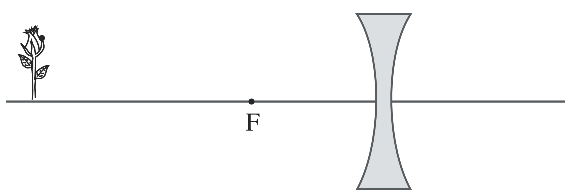
7. A flower is placed before a converging lens as shown in the figure below. The image produced by the lens is

A. virtual, upright, diminished
B. real, upright, enlarged
C. real, inverted, diminished
D. virtual, inverted, enlarged



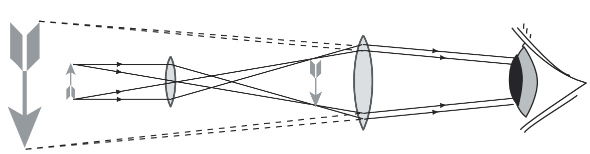
8. A flower is placed before a diverging lens as shown in the figure below. The image produced by the lens is

A. virtual, upright, diminished
B. real, upright, enlarged
C. real, inverted, diminished
D. virtual, inverted, enlarged



9. What type of optical instrument is represented by the ray diagram below?

A. compound microscope
B. simple magnifier
C. astronomical telescope
D. slide projector



10. A converging lens has a focal length of F. An object is placed at distance F from the lens on the axis. The image formed is

A. F distance from the lens.
B. 2F distance from the lens.
C. between F and 2F distance from the lens.
D. at infinity.

11. An object is placed at a distance of 8 cm from a 25 diopter positive lens. The magnification of the image is

A. –0.32
B. –1.0
C. +32
D. +200

12. Examining the scene in the conservatory, the detective holds the magnifying glass 20cm over the candlestick, magnifying a fingerprint 5×. What is the focal length of the magnifier?

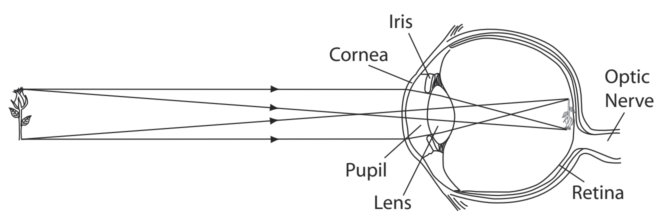
A. 4 cm
B. 10 cm
C. 25 cm
D. 100 cm

13. Chromatic aberration is a common problem with

A. convex mirrors.
B. lenses made with homogeneous glass.
C. lenses made with layers of salt glass and crown glass.
D. concave mirrors.

The following passage pertains to questions # - #.

The total optical power of the relaxed human eye is approximately 60 diopters. The cornea accounts for approximately two thirds of this refractive power and the crystalline lens contributes the remaining third. In focusing, the ciliary muscle contracts to reduce the tension or stress transferred to the lens by the suspensory ligaments. This results in increased convexity of the lens which in turn increases the optical power of the eye. As humans age, the amplitude of accommodation reduces from approximately 15 to 20 diopters in the very young, to about 10 diopters at age 25, to around 1 diopter at 50 and over.



The fact that optical powers are approximately additive enables an optometrist to prescribe corrective lenses as a simple correction to the eye's optical power, rather than doing a detailed analysis of the entire optical system (the eye and the lens).

However, the design of a proper optical treatment may be complicated if the patient suffers from astigmatism. Astigmatism is an optical defect in which vision is blurred due to the inability of the optics of the eye to focus a point object into a sharp focused image on the retina. This may be due to an irregular or toric curvature of the cornea or lens. The two types of astigmatism are regular and irregular. Irregular astigmatism is often caused by a corneal scar or scattering in the crystalline lens, and cannot be corrected by standard spectacle lenses, but can be corrected by contact lenses. The more common regular astigmatism arising from either the cornea or crystalline lens can be corrected by eyeglasses or toric lenses. A 'toric' surface resembles a section of the surface of a football where there are two regular radii, one smaller than the other one. This optical shape gives rise to astigmatism in the eye.

The refractive error of the astigmatic eye stems from a difference in degree of curvature refraction of the two different meridians (i.e., the eye has different focal points in different planes). For example, the image may be clearly focused on the retina in the horizontal plane, but not in the vertical plane. Astigmatism causes difficulties in seeing fine detail resulting in blurred vision. Three options exist for the treatment of astigmatism: spectacles, contact lenses (either hard contact lenses or toric contact lenses), and refractive surgery.

14. According to the information in the passage which of the following represents an approximate value of the focal length of the relaxed human eye?

A. 6 mm
B. 15 mm
C. 24 mm
D. 30 mm

15. Hyperopia, also known as farsightedness, is often caused by

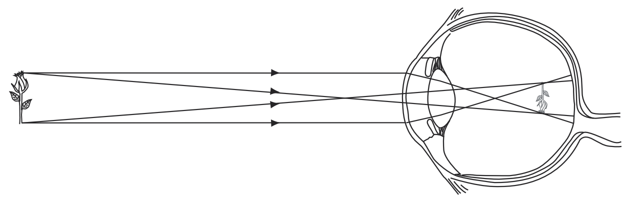
A. a cornea that is too rounded.
B. edematous swelling of the lens.
C. excessive contraction of the ciliary body.
D. an eye that is too short.

16. Ciliary muscle contraction

A. decreases the refractive power of the lens.
B. permits the eye to focus on more distant objects.
C. increases the radius of curvature of the lens.
D. allows the lens to obtain a more rounded shape.

17. The figure below shows an eye that suffers from the condition of

A. presbyopia
B. myopia
C. hyperopia
D. astigmatism



18. If a person with normal vision looked through a pair of glasses designed to correct astigmatism where the curvature of the horizontal meridian of the eye is too great (like a football lying on its side), they would see

A.

B.

C.

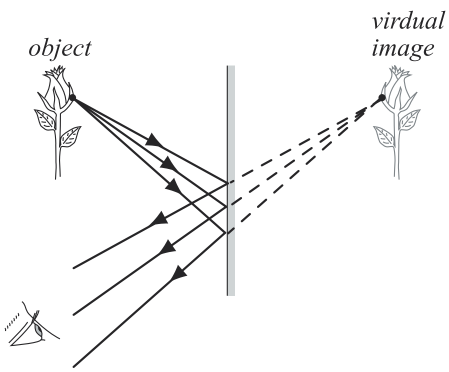
D.

19. Typical glasses for mild myopia will have a power of

A. −1.0 to −3.0 diopters
B. 1.0 to 5.0 diopters
C. 10 to 20 diopters
D. 20 to 60 diopters

Geometric Optics
Answers and Explanations

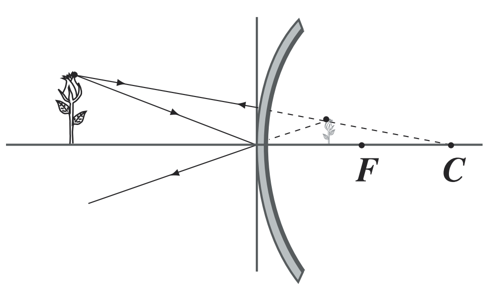
1. D
A plane mirror creates a virtual image, located behind the mirror. With a virtual image, the rays of light do not intersect at the image point. The rays of light appear to originate from the image, diverging from the image point.



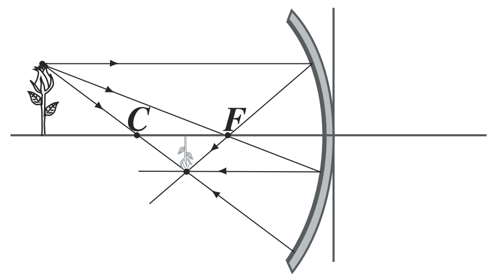
2. A
The magnification of the image produced by a lens or mirror is determined using the following formula where I is the distance of the image along the optical axis from the optical device and O is the object distance.

$$M = \frac{-I}{O}$$

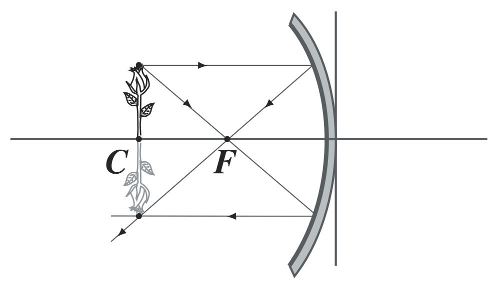
If magnification is positive, the image is upright. If the absolute value of the magnification is less than 1, the image is diminished. In other words, with a magnification of 0.7, the optical device is producing an image which is diminished, upright, and virtual. This is the type of image produced by either a diverging lens or a convex mirror.



3. C
If the object distance with a concave mirror is greater than the center of curvature of the mirror, C (which also equals $2F$), the image produced will be real, inverted, and diminished.



4. A
If the object distance with a concave mirror is equal to the center of curvature of the mirror, C (which also equals $2F$), the image produced will be real, inverted, and same sized. The image distance will equal C.

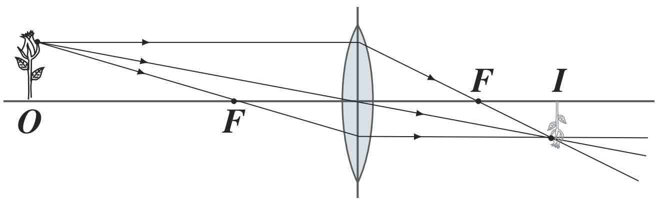


5. B
Although there is variety in the shape, converging lenses are thicker in the middle and thin at their upper and lower edges. Converging lenses have a positive focal length.

6. A
Diverging lenses are thinner in the middle. Diverging lenses have a negative focal length.

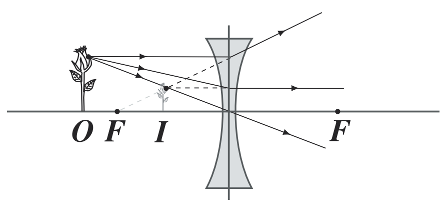
7. C

If the object distance with a converging lens is greater than $2F$, the image produced will be real, inverted, and diminished.



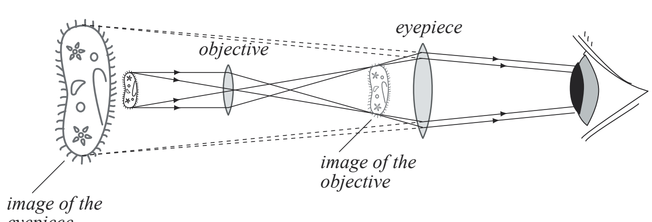
8. A

The image produced by a single diverging lens is always diminished, upright and virtual (DUV).



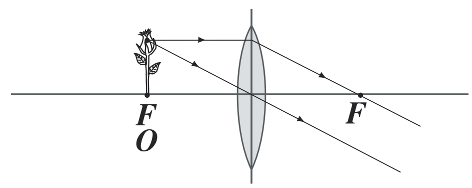
9. A

To understand systems of two or more lenses such as the compound microscope, take it one step at a time. The image of the first lens serves as the 'object' of the second. A compound microscope consists of two converging lenses, an objective lens and an eyepiece lens. The object is situated just beyond the focal length of the objective lens, producing a real inverted image. This image serves as the 'object' for the eyepiece lens, just within its focal length. The eyepiece produces a enlarged virtual image of the inverted real image of the objective.



10. D

When the distance of the object from a converging lens is equal to the focal length, no image is formed.



11. C

The lens equation relates the focal length, image distance, and object distance for an optical device.

$$\frac{1}{F} = \frac{1}{I} + \frac{1}{O}$$

Alternatively, we can express $1/F$ as the power of the lens, D, in diopters (m^{-1}). The lens equation is often expressed:

$$D = \frac{1}{I} + \frac{1}{O}$$

For our problem then:

$$25\ m^{-1} = \frac{1}{I} + \frac{1}{.08\ m}$$

$$25\ m^{-1} = \frac{1}{I} + 12.5\ m^{-1}$$

$$\frac{1}{I} = 12.5\ m^{-1}$$

$$I = .08\ m$$

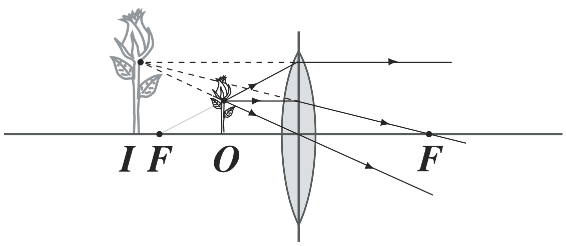
To determine magnification:

$$M = \frac{-I}{O} = \frac{-.08m}{.08m} = -1$$

Note that with a power of 25 diopters, our lens has a focal length of 4cm. When the object distance is equal to $2F$ with a converging lens (positive focal length), the image is real, inverted and same sized.

12. C

A magnifying glass is a converging lens. To observe an object, you position a magnifying glass above the object so that the object is nearer to the lens than the focal length of the lens. For an object placed nearer to the converging lens than the focus, the image produced is enlarged, upright, and virtual.



In our problem, the magnifier is placed 20cm above the object, and it is magnified 5×. This corresponds to an image distance of -100cm. (A negative image is a virtual image).

$$M = 5 = \frac{-I}{O} = \frac{-1.0\text{ m}}{.20\text{ m}}$$

Knowing the image distance for a given object, we can determine the focal length of the magnifying glass to be 25cm.

$$\frac{1}{F} = \frac{1}{I} + \frac{1}{O}$$

$$\frac{1}{F} = \frac{-1}{1.0\text{ m}} + \frac{1}{.20\text{ m}}$$

$$F = .25\text{ m}$$

13. B

Chromatic aberration is a failure of a lens to focus all colors to the same point. It is caused by dispersion, ie. the refractive index of the lens varies with the frequency of light. Chromatic aberration manifests itself as "fringes" of color along boundaries within the image. Chromatic aberration can be minimized by assembling a compound lens from materials with differing dispersion properties. The most common type is an achromatic doublet, with elements made of crown and flint glass.

14. B

The optical power of the relaxed human eye is approximately 60 diopters (m^{-1}), a fact presented in the passage. The focal length of a lens is the reciprocal of its power in diopters.

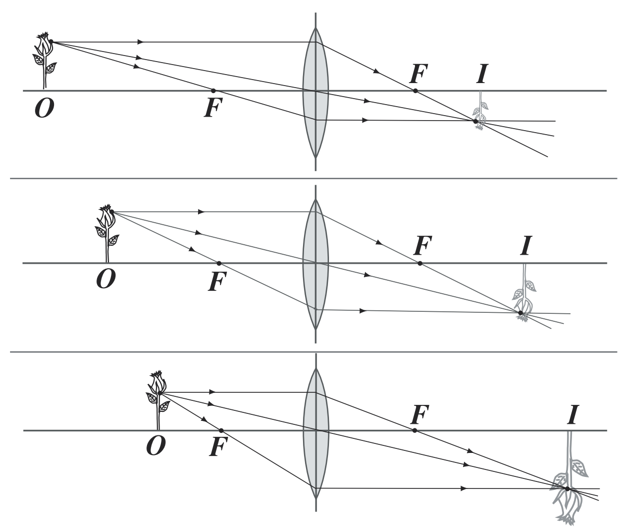
$$D = \frac{1}{F}$$

$$60\text{ m}^{-1} = \frac{1}{F}$$

$$F = \frac{1}{60}\text{ m} \sim 15\text{ mm}$$

15. D

An individual with hyperopia has difficulty seeing near objects. To understand hyperopia, it's very helpful to visualize what occurs with image distance (the real inverted image formed by a converging lens) as the object is moved closer to the lens. Notice as the object is moved closer, the image moves further away from the lens on the other side.



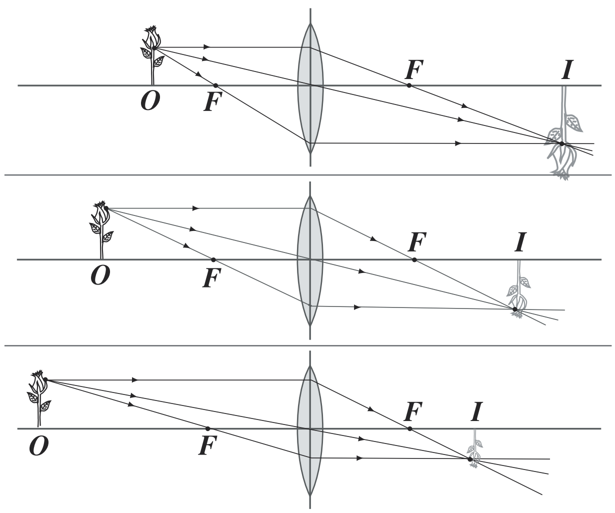
The cause of hyperopia is often that the eyeball is too short. In other words, the distance between the refractive elements (corena and lens) and the retina is too short, so for near objects the focused real image would be behind the retina. Accommodation is not sufficient to shorten the focal length of the eye and pull the image forward onto the retina. Light that originated from the same object point instead is landing at different locations on the retina. The result is blurry vision.

16. D

Contraction of the ciliary muscle reduces the tension on the fibers of the ciliary zonule and causes the lens to curve or become more spherical. This increases the optical power of the eye, accommodating for near vision.

17. D

An individual with myopia has difficulty seeing far objects. To understand myopia, it's very helpful to visualize what occurs with image distance as the object is moved further away from the lens. Notice as the object is moved further away, the image moves closer to the lens on the other side.



The cause of myopia is often that the eyeball is too long (or the eye has too short a focal length). For far objects, the retina would need to be further in for a focused image to land on it. Light that originated from the same object point instead is landing at different locations on the retina. The result is blurry vision for far objects.

18. C

To repair the astigmatism, the corrective lenses adjust the focal length in the horizontal plane to match the vertical focal length. To a person with normal vision the glasses would appear out of focus in the horizontal plane but not in the vertical plane.

19. A

Myopia or nearsightedness is the inability of the eye to focus on distant objects. It is the result of either a bulging cornea or too long an eyeball. With myopia the image distance for distant objects is in front of the retina.

The cure for the nearsighted eye is to equip it with a diverging lens. This spreads the light rays, shifting the focal length further back, so that the image will land on the retina. A diverging lens has a negative focal length, so its power in diopters will likewise be negative. −1.0 to −3.0 diopters is the range of the typical prescription for mild myopia.

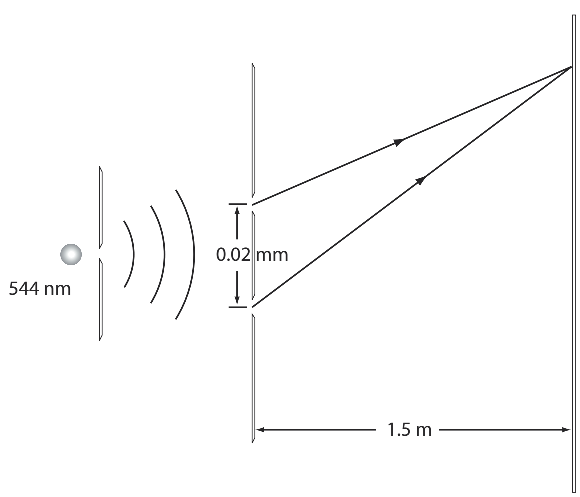
Wave Optics Practice Items

1. Light has the property of being able to bend around corners. This phenomenon is known as

A. diffraction
B. refraction
C. dispersion
D. polarization

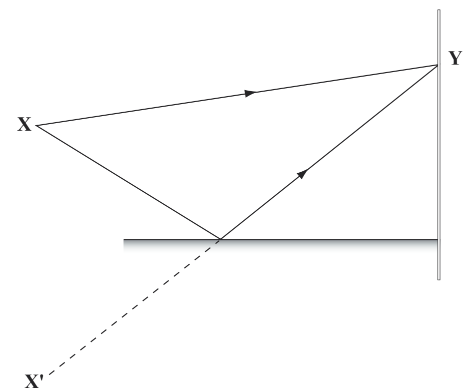
2. The figure below shows light from a helium-neon laser passing through two slits before striking a screen. Compared to the light from the upper slit, how much further does the light from the lower slit travel to reach a point on the screen at an angle 30° above the two slits?

A. 2.7×10^{-7} m
B. 1.0×10^{-5} m
C. 7.5×10^{-5} m
D. 7.5×10^{-1} m



3. A light source is placed at point **X** near a mirror as shown in the figure below. Light rays of wavelength λ can reach the screen either by a direct path or through reflection. An interference pattern is observed on the screen similar to the pattern observed in Young's interference. Which of the following descries the condition for a *dark* fringe at point **Y**?

A. $|X'Y| - |XY| = n\lambda \quad (n = 1,2,3\ \ldots)$
B. $|X'Y| - |XY| = (n + \tfrac{1}{2})\lambda \quad (n = 1,2,3\ \ldots)$
C. $|X'Y| - |XY| = n\lambda \quad (n = 1,3,5\ \ldots)$
D. $|XX'|^2 + |XY|^2 = |X'Y|^2$



4. What is the minimum thickness of a soap bubble where constructive interference occurs when illuminated by a 589 nm sodium lamp?

A. 147 nm
B. 295 nm
C. 589 nm
D. 1.18 μm

5. A coin is suspended halfway between a monochromatic light source and a screen. Circular fringes are observed near the shadow's edge. What is observed in the center of the shadow?

A. an inverted image of the coin
B. an iridescent circular pattern
C. a cross-hatch pattern of bright fringes
D. a bright spot

6. What would happen if the monochromatic light source in Young's double slit experiment were replaced with white light?

A. No diffraction would occur.
B. The central fringe would be violet and the outermost fringes would be red.
C. A more widely spaced pattern of white fringes would appear.
D. The bright central fringe would be white and the other fringes would be colored.

7. Approximately 50 μm is the minimum separation between two point sources the normal human eye can distinguish at the near point. This is approximately equal to the thickness of a human hair. Why can't object points nearer than 50 μm from each other be distinguished by the human eye?

A. The inverted real image would land behind the retina.
B. Too many orders of diffracted light would be captured to separate the object points.
C. The two points could be distinguished if moved further away than the near point.
D. The central maxima of the airy disk images of the two object points would overlap.

8. The first diffraction order was observed at a 30° angle to the surface when incident X-rays having a wavelength of 2.65×10^{-10} m were reflected off of the surface of a NiS crystal. The reflected rays were visualized photographically. What is the spacing between atomic layers in this crystal?

A. 1.33 Å
B. 2.65 Å
C. 3.75 Å
D. 5.30 Å

9. If one places a block of a birefringent material such as calcite onto a sheet of paper with an image, one sees two images through the block. If the two images are then viewed through a rotating sheet of polaroid film, the two images

A. alternately appear and disappear.
B. merge into a single image.
C. will be inverted.
D. will be magnified.

10. Transmission of plane polarized light through a solution of pure optical isomer results in rotation of the optical axis of the transmitted light. The degree of observed rotation may be measured by a polarimeter. The rotation in degrees observed upon passing polarized light through a path length of 1 decimeter (dm) at a concentration of 1 g/mL is known as the

A. circular birefringence
B. dextrarotation
C. specific rotation
D. chirality

11. UV circular dichroism spectroscopy is primarily used within life sciences research to investigate

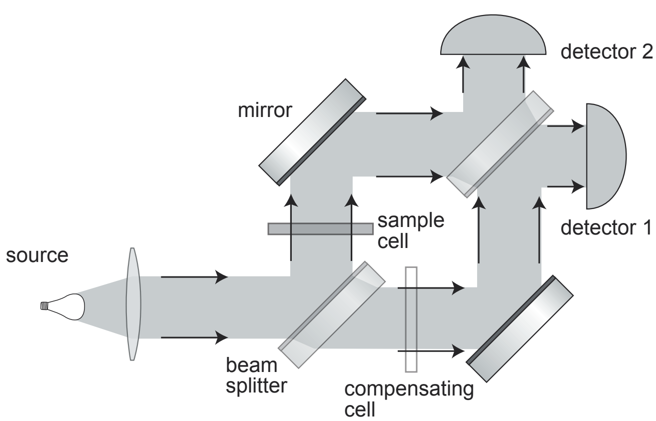
A. the secondary structure of proteins.
B. the kinetics of enzymatic reactions.
C. the crystal field splitting energy of metallic cofactors.
D. the UV absorbance of biological pigments.

The following pertains to questions 12 - 16.

The Mach–Zehnder interferometer is used to determine the relative phase shift variations between two collimated beams derived by splitting light from a single source. The interferometer has been used, among other things, to measure phase shifts between the two beams caused by a change in the optical path length of one of the beams by the introduction of a sample.

A collimated beam is split by a half-silvered mirror. The two resulting beams (the "sample beam" and the "reference beam") are each reflected by a mirror. The two beams then pass a second half-silvered mirror and enter two detectors. Phase change occurs for a reflection when a wave reflects off a boundary from low to high refractive index but not when it reflects off a boundary from high to low.

The fully silvered and half-silvered surfaces of all mirrors, except the last, face the inbound beam, and the half-silvered surface of the last mirror faces the outbound beam exiting in the same orientation as the collimated original beams.



The Mach–Zehnder interferometer is a more versatile instrument than the Michelson interferometer. Each of the well separated light paths is traversed only once, and the fringes can be adjusted so that they are localized in any desired plane. Typically, the fringes would be adjusted to lie in the same plane as the test object, so that fringes and test object can be photographed together.

12. The Mach–Zehnder interferometer works because when two waves originating from a single source recombine a pattern results determined by

A. the difference in frequency between the waves
B. the phase difference between the waves
C. the refractive index of the beam splitter
D. the difference in wavelength between the waves

13. Light traveling in test and reference beams of equal optical path through a Mach–Zehnder interferometer leads to which result on detector 2?

A. a bright pattern
B. no image
C. incoherence in the transversal direction
D. a horizontally inverted image

14. A sample is introduced into the sample cell in which heat transfer and convection currents produce varying indices of refraction throughout the sample.

A. A fringe pattern appears on both detectors.
B. No results may be recorded.
C. Dispersion of the monochromatic source occurs.
D. Detector 2 receives polarized light.

15. What is a likely to result with a white light source with an empty sample cell if the compensating cell is not also included?

A. fringes of varying color on detector 1
B. fringes of varying color on detector 2
C. fringes of varying color on both detectors
D. no interference patterns

16. A Mach-Zehnder interferometer is illuminated by light of λ_{vacuum} = 500 nm ($\tilde{\nu}$ = 20,000 cm^{-1}). The interferometer contains a 1 cm gas-filled sample cell. As the gas is evacuated from the cell, 8 fringes cross a point in the field of view of detector 1. The refractive index of the gas at its original concentration is closest to which of the following values?

A. 1.04
B. 1.004
C. 1.0004
D. 1.00004

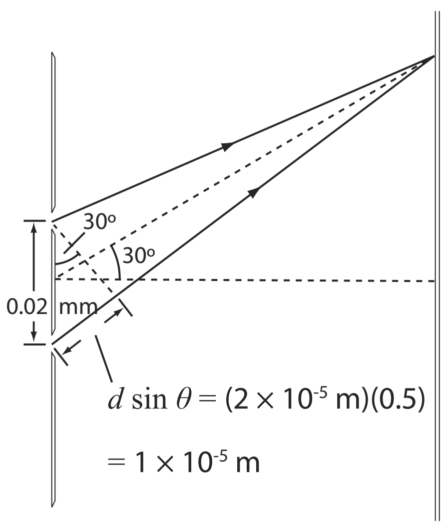
Wave Optics
Answers and Explanations

1. A

Diffraction is the bending of waves around the corners of an obstacle or through an aperture into the region of geometrical shadow of the obstacle. The diffracting object or aperture effectively becomes a new source of the propagating wave. Diffraction can be understood in terms of the Huygens principle that treats each point in a propagating wave-front as a collection of individual spherical wavelets. The characteristic interference pattern resulting from diffraction is most pronounced when light from a coherent, monochromatic source (such as a laser) encounters a slit/aperture that is comparable in size to its wavelength.

2. B

The extra distance from the lower slit is the product of the spacing between the two slits, d, and the sine of the angle, θ, to the point on the screen mentioned in the passage.

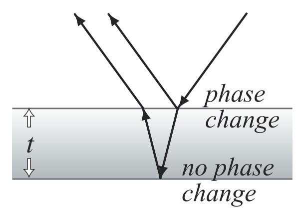


3. B

$|X'Y| - |XY|$ equals the extra distance traveled by the reflected ray. One might think that a bright fringe occurs if this extra distance is an integral number of wavelengths ($n\lambda$), but remember that when light reflects within a fast medium off of a high index of refraction surface it undergoes a 180° phase change, so we need to offset the condition for constructive interference by half a wavelength.

4. A

In thin film interference, light reflected from the front and rear surfaces of a thin film combines to form a resultant wave. Whether constructive or destructive interference occurs depends on whether the combining rays are in or out of phase. The phase difference of the rays depends on the wavelength of light in the film medium, the thickness of the film (assume the light rays are nearly normal to the surface), and whether or not there are any phase changes with either reflection. Hard reflection is reflection off of a medium of higher index of refraction and leads to a 180° change of phase. Soft reflection is reflection off of a medium of lower index of refraction and does not produce a change of phase.



Normally, in interference problems, a path difference (2t with thin films) equal to an integral number of wavelengths produces constructive interference, but if one of the reflections is hard and the other soft the result is destructive interference.

Condition of Constructive Interference (with one reflection having a phase change)

$$2t = \left(m + \tfrac{1}{2}\right)\lambda_n \quad (m = 0,\ 1,\ 2, \ldots)$$

In other words, if the thickness of the soap film is one fourth the wavelength of the incident light, 147nm, the path difference would be half the wavelength. Because one of the reflections resulted in a 180° change of phase, this produces constructive interference.

5. D

Diffraction leads to bending of the light around the coin into the region of its shadow. The edges of the coin effectively becomes a new source of the propagating light. The diffracting rays from different points along the entire edge circumference will have traveled exactly the same distance to the very center of the coin, so they will be in phase there and produce a bright spot resulting from constructive interference.

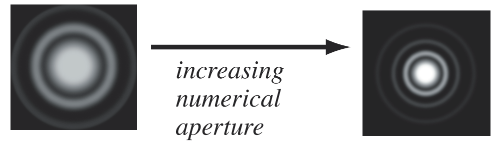
6. D

Because there is no path difference from either slit to the position of the central fringe, all of the variously colored components of white light will be in phase in the center. Elsewhere on the screen, however, the phase difference of the light rays from the two slits will depend on the wavelength, so at a certain angle only a certain wavelength will be in phase while others will be undergoing destructive interference.

7. D

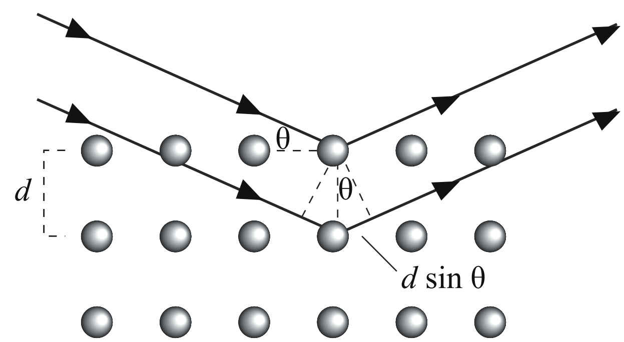
Diffraction of light passing through a circular aperture produces an interference pattern similar to single slit diffraction. By Huygen's principle, each portion of the aperture acts as a source of waves. For a given image point, the interference of wavelets yields a diffraction pattern known as an airy disk. Widening the slit narrows the central maximum in single slit diffraction. Likewise, widening a circular aperture increases the phase difference possible for different light paths, increasing the number of diffraction orders captured, which decreases the size of the central maxima of the airy disk image of a given point source.

When the central maximum of one airy disk falls on the first minimum of another (Rayleigh's criterion), the images are said to be just resolved. If the width of the pupil were greater, the human eye could resolve finer detail. This would increase the number of diffraction orders captured and decrease the size of the central maxima of object point airy disc patterns. Decreasing the size of central maxima increases the ability to resolve two airy disk patterns.



8. B

The basic principles underlying X-ray crystallography are similar to thin film interference. It can be seen in the figure below that the path length difference for light reflecting off of the top layer of the crystal and the adjacent layer below it is $2d \sin \theta$.



Bragg's Law describes how constructive interference will be observed if this path length difference is equal to an integral number of wavelengths of the incident light.

$$2d \sin \theta = n\lambda \qquad (n = 1,2,3 \ldots)$$

The first diffraction order would at an incident angle of 30° if the crystal layer spacing exactly equaled the wavelength of the incident X-rays.

$$2d \sin 30^\circ = (1)\, \lambda$$

$$d = \lambda$$

9. A

In birefringent materials, such as calcite and quartz, the index of refraction is not the same in all directions. Double refraction causes an unpolarized light beam to be split into an ordinary (O) ray and an extraordinary (E) ray, which are polarized in mutually perpendicular directions. The two images seen when the block is placed on the paper are produced by these two rays respectively.

A polaroid film only allows the components of the electric field vibrations to pass that are parallel to its transmission axis. Because the ordinary and extraordinary rays are polarized in mutually perpendicular directions, the rotating polaroid film will allow a varying intensity of transmission of each which is phase shifted in the rotation. In other words, the two images will alternately appear and disappear.

10. C

Specific rotation is common standard for optical rotation. It allows us to compare samples collected under different concentrations and path lengths.

observed rotation

$$[\alpha] = \frac{\alpha_{observed}}{c \times l}$$

specific rotation

concentration g/ml

path length dm

11. A

Circular dichroism spectroscopy is based on the differential absorption of left-hand and right-hand circular polarized light. Circularly polarized light occurs when the direction of the electric field vector rotates about its propagation direction. At a single point in space, the circularly polarized-vector will trace out a circle over one period of the wave frequency.

The far-UV CD spectrum of a protein can reveal information about the secondary structure of the protein. The technique can be used to estimate the fraction of the protein in the alpha-helix or beta-sheet conformations, for example.

12. B

A basic scenario repeats itself throughout wave optics from Young's interference, thin layer interference, to the Michelson or Mach-Zehnder interferometers. Light which was originally in phase and traveling together is separated in some way, whether by diffracting through separate slits (or different points in the same aperture in single slit diffraction), reflecting off different boundaries in a thin layer, or being split by a beam splitter to travel different paths in interferometry. The rays then recombine. Interference results. The different optical paths they have followed may or may not have produced a phase difference leading to either constructive or destructive interference.

13. B

At detector 2, in the absence of a sample, the sample beams and reference beams will arrive with a phase difference of half a wavelength, yielding complete destructive interference. When light traveling in a fast medium reflects off a boundary to a slow medium, the light undergoes a 180° phase shift. This is known as a hard reflection. The reference beams arriving at detector 2 will have undergone a 180° phase shift of due to one hard reflection. The sample beams arriving at detector 2 will have undergone two hard reflections. Therefore, when there is no sample, only detector 1 receives light.

14. A

An alteration of optical path occurs as the collimated sample beams travel through the sample. The collimated beams are variously altered across the wave front. When they recombine with the reference beam, the phase shift differences create an image of the sample as an interference pattern on both detectors. Because both the sample and reference beams on their paths to detector 1 undergo two hard reflections, while on their paths to detector 2 undergo two and one hard reflections respectively, there is a 180° offset in the phase shift relationships reflected in the images on the two detectors, so they will appear as negative images of each other.

15. C

The collimated beams of white light passing through the glass of the sample cell will have undergone dispersion. Dispersion occurs in glass because the phase velocity of light in glass varies slightly with the frequency of light, ie. the index of refraction in glass is slightly different for the different colors. Without the compensating cell present in the path of the reference beam, the dispersion occurring in the sample cell will cause the optical path of the sample beam to slightly different for each frequency. This will lead to a pattern of colored fringes in each detect

16. C

As the gas is being evacuated from the sample cell, the index of refraction within is steadily decreasing from the value for the gas at its original concentration to the vacuum value of 1. As the index of refraction decreases, the wavelength of the light within the sample cell increases. For light of a given frequency, the faster the medium the lower the index of refraction and the longer the wavelength.

$$\frac{v_2}{v_1} = \frac{n_1}{n_2} \qquad \frac{\lambda_2}{\lambda_1} = \frac{v_2}{v_1} \qquad \frac{\lambda_2}{\lambda_1} = \frac{n_1}{n_2}$$

As the wavelength in the sample cell increases, eight phase shift cycles occur in the interference of the sample and reference beams. In other words, it required eight more wavelengths to cross the sample cell containing the gas than the vacuum. Those eight extra wavelengths are subtracting from the optical path of the sample beam as the sample cell is being evacuated.

A helpful clue in the question stem might make the problem easier to conceptualize and solve. In addition to presenting the wavelength in nm, it also provides the same value in the form of wavenumber, $\tilde{v}$. The wavenumber is the reciprocal of the wavelength. A value of 20,000 cm^{-1} tells you that there are 20,000 cycles per centimeter for this particular frequency of light in a vacuum. The sample cell itself is 1 cm long. The wavenumber of the light in the gas was therefore 20,008 cm^{-1}. Because wavenumber is the reciprocal of the wavelength, which is inversely proportional to the index of refraction, wavenumber is directly proportional to index of refraction.

$$\frac{n_{\text{gas}}}{n_{\text{vacuum}}} = \frac{2.0008 \times 10^4 \text{ cm}^{-1}}{2.0 \times 10^4 \text{ cm}^{-1}}$$

$$\frac{n_{\text{gas}}}{1} = \frac{2.0 \times 10^4 + 8}{2.0 \times 10^4}$$

$$n_{\text{gas}} = 1 + \frac{8}{2.0 \times 10^4}$$

$$n_{\text{gas}} = 1 + 4.0 \times 10^{-4} = 1.0004$$

Modern Physics Practice Items

1. Often observed in radio astronomy at a characteristic frequency of 1420.41 MHz, the precession frequency of neutral hydrogen atoms, the microwaves of the hydrogen line come from the atomic transition of an electron between the two hyperfine levels of the hydrogen 1 s ground state that have an energy difference of ≈ 5.87μeV. What is the wavelength in a vacuum of this electromagnetic radiation?

A. 4.1×10^{-15} m
B. 21 μm
C. 21 cm
D. 4.7 m

2. What is the energy carried by a single photon of yellow light, $\lambda = 505$ nm ($h = 6.63 \times 10^{-34}$ J·s)?

A. 3.5×10^{-40} J
B. 3.9×10^{-19} J
C. 7.6×10^{-12} J
D. 1.2×10^{-10} J

3. The same X-rays that can eject photoelectrons from zinc metal may also be diffracted by the surface of ZnS crystal. Which of the following principles does this demonstrate?

A. Electrons may be diffracted.
B. Zinc metal exhibits Rayleigh scattering of incident X-rays while ZnS does not.
C. The work function for a pure metal is higher than it is for an ore of the metal.
D. Light exhibits both particle and wave properties.

4. Consider two monochromatic sources of light A and B. The frequency of the electromagnetic waves emitted by source A and the power of source A are both half that of source B. In the time it takes source A to emit *n* number of photons, how many photons does source B emit?

A. ¼ *n*
B. *n*
C. 2*n*
D. 4*n*

5. A 100 W HeNe laser emits coherent yellow light ($\lambda = 598$ nm). Assuming light production were 100% efficient, how many photons does it produce per second?

A. 6.0×10^{4}
B. 3.0×10^{20}
C. 6.0×10^{23}
D. 2.5×10^{41}

6. What will be the result of measuring the distance from a hydrogen electron in its ground state to the nucleus?

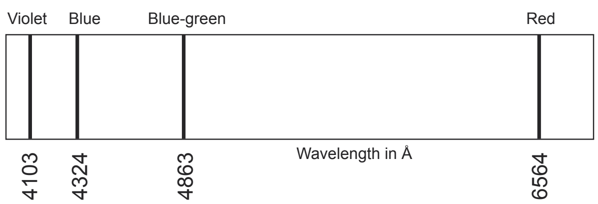
A. the Bohr radius
B. a set of distances corresponding to the Balmer series of spectral emissions
C. a distance equal to the radius of the nucleus
D. the Bohr radius, most likely, but a range of distances is possible

7. A spacecraft exposed to sunlight will develop a positive charge. This is most likely due to

A. the photoelectric effect
B. pair production
C. quantum tunneling
D. the solar wind

8. Below is a portion of the line spectrum in the visible region for hydrogen. The red line results from photons released in the electronic transition from n = 3 to n = 2. What is the value of the energy involved in this electronic transition?
(h = 6.63 × 10^{-34} J·s)

A. 3.01 × 10^{-20} J
B. 1.00 × 10^{-19} J
C. 3.01 × 10^{-19} J
D. 4.84 × 10^{-19} J



9. In a 1923-27 experiment by Clinton Davisson and Lester Germer, electrons, scattered by the surface of a crystal of nickel metal, displayed a diffraction pattern. This confirmed the hypothesis, advanced by Louis de Broglie in 1924, of

A. quantized atomic energy levels
B. the uncertainty principle
C. wave-particle duality
D. photon theory

10. Which of the following particles would have the shortest de Broglie wavelength traveling at 1.5 × 10^4 m/s?

A. neutrino
B. positron
C. proton
D. α particle

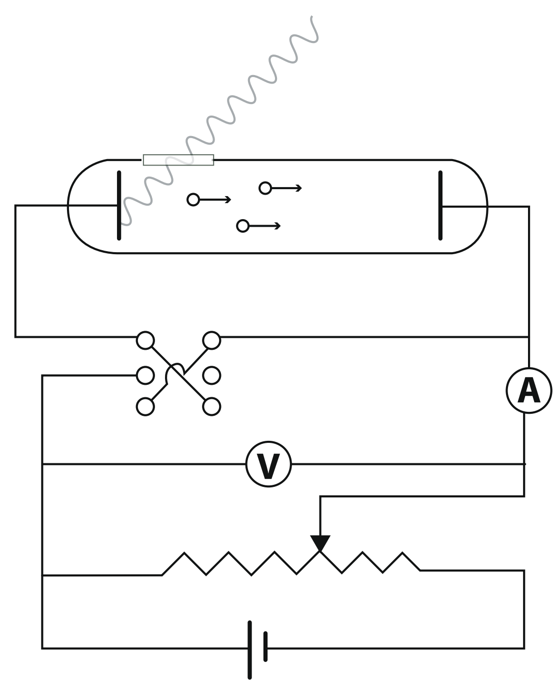
The following passage pertains to questions 11 - 15.

The photoelectric effect is the emission of electrons when electromagnetic radiation strikes a material. Electrons emitted in this manner can be called photoelectrons. In 1905, Einstein proposed an explanation of the photoelectric effect using a concept first put forward by Max Planck that light waves consist of tiny bundles or packets of energy known as photons or quanta.

The maximum kinetic energy of an ejected electron is given by

$$K_{mzx} = hf - \phi$$

The term φ is the work function which gives the minimum energy required to remove an electron from the surface of the metal.



The relation between current and applied voltage in the apparatus above illustrates the nature of the photoelectric effect. A light source illuminates a metal plate within a vacuum tube, and another plate electrode collects any emitted electrons. The potential between the two plates is varied and the current flowing in the external circuit between the two plates is measured.

If the frequency and the intensity of the incident radiation are fixed, the photoelectric current increases gradually with an increase in the positive potential on the collector electrode until all the photoelectrons emitted are collected. The photoelectric current attains a saturation value and does not increase further for any increase in the positive potential. The saturation current increases with the increase of the light intensity. It also increases with greater frequencies of incident light.

If we apply a negative potential to the collector plate Q with respect to the plate P and gradually increase it, the photoelectric current decreases, becoming zero at a certain negative potential. The negative potential on the collector at which the photoelectric current becomes zero is called the stopping potential.

11. If the device illuminating the metal is changed from a lamp producing mid UV light to a far UV lamp, the speed of emitted photoelectrons will

A. remain the same.
B. increase.
C. decrease.
D. become zero.

12. For a given frequency of incident radiation leading to production of photoelectrons in the apparatus depicted in the passage, the stopping potential

A. is determined by the minimum kinetic energy of the photoelecrons that are emitted.
B. decreases when a metal with lower threshold frequency is bombarded with the light.
C. increases with decreasing wavelength of incident light.
D. is independent of the intensity of the radiation.

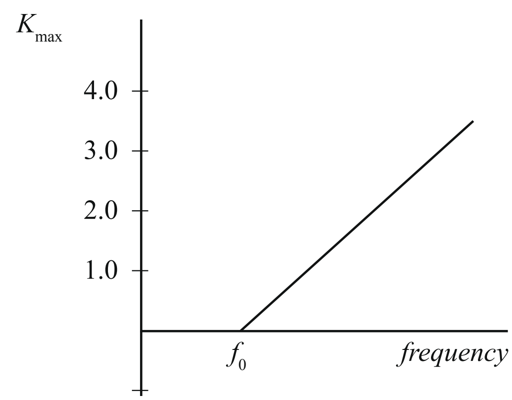
13. The graph below shows current measurements for a photoelectric effect trial with a constant intensity of varying frequencies of incident UV radiation on a 1.5 cm^2 copper plate. A positive potential was applied to the collector plate. Which is the correct way to compute the value of the work function for copper?

A. $(6.63 \times 10^{-34}\ \text{J·s})(1.1 \times 10^{15}\ \text{s}^{-1})$

B. $\dfrac{(6.63 \times 10^{-34}\ \text{J·s})(3 \times 10^{8}\ \text{m/s})}{(1.1 \times 10^{15}\ \text{s}^{-1})}$

C. $9.0 \times 10^{-19}\ \text{J} - (6.63 \times 10^{-34}\ \text{J·s})(1.1 \times 10^{15}\ \text{s}^{-1})$

D. $\dfrac{(6.63 \times 10^{-34}\ \text{J·s})(1.1 \times 10^{15}\ \text{s}^{-1})}{2\pi}$

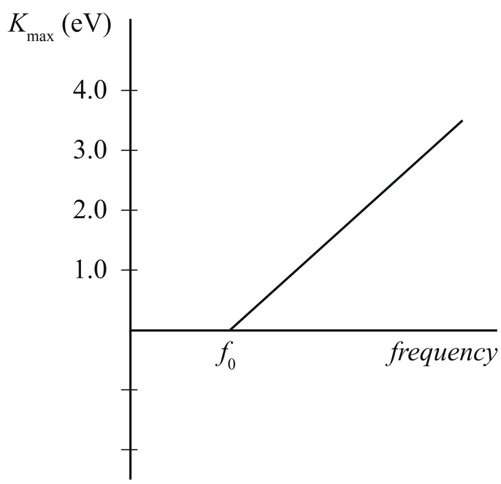


14. A trial was conducted using the apparatus depicted in the passage. A 145-nm light (*photon energy* = 8.55 eV) was shined on an unknown metal. The measured photocurrent dropped to zero at potential – 3.50 V. Determine the maximum kinetic energy possessed by the photoelectrons emitted from the metal surface.

A. 3.50 eV
B. 5.05 eV.
C. 8.55 eV.
D. 12.05 eV

15. The graph below represents a series of photoelectric effect trials employing a particular metal. Which of the following represents the work function of the metal?

A. 1.50 eV
B. 2.50 eV.
C. 3.00 eV.
D. 3.50 eV



Modern Physics
Answers and Explanations

1. C

To find the wavelength, divide the frequency into the wave speed.

$$\lambda = \frac{c}{f}$$

$$\lambda = \frac{3 \times 10^{8}\,\text{m/s}}{1.4 \times 10^{9}\,\text{s}^{-1}}$$

$$\lambda = 2.1 \times 10^{-1}\,\text{m}$$

Always let a problem be easy if it wants to be. Don't let the hand-waving with technical details convince you a problem is harder than it is.

2. B

Photon energy equals Planck's constant times the frequency.

Because our answer choices are very widely spaced, we can give ourselves plenty of allowance for mental math.

$$E = hf$$

$$E = \frac{hc}{\lambda}$$

$$E = \frac{(6.63 \times 10^{-34}\,\text{J·s})(3 \times 10^{8}\,\text{m/s})}{(5.05 \times 10^{-7}\,\text{m})}$$

$$E \sim \frac{(7 \times 10^{-34}\,\text{J·s})(3 \times 10^{8}\,\text{m/s})}{(5 \times 10^{-7}\,\text{m})}$$

$$E \sim 4 \times 10^{-19}\,\text{J}$$

3. D

The photoelectric effect exemplifies the particle (photon) nature of light. Diffraction exemplifies the wave nature of light.

4. B

Photon energy equals Planck's constant times the frequency.

$$E = hf$$

If source A emits light having half the frequency of source B, the energy of its photons must be half the energy of the photons of source B.

Power is the rate of energy expenditure. If the individual photons of source A have half the energy of those of source B, and source A is operating at half the power of source B, it directly follows that the two sources are emitting photons at the same rate.

5. B

First we need to determine the energy per photon. Photon energy equals Planck's constant times the frequency.

$$E = hf$$

$$E = \frac{hc}{\lambda}$$

$$E = \frac{(6.63 \times 10^{-34}\,\text{J·s})(3 \times 10^{8}\,\text{m/s})}{(5.98 \times 10^{-7}\,\text{m})}$$

$$E \sim \frac{(7 \times 10^{-34}\,\text{J·s})(3 \times 10^{8}\,\text{m/s})}{(6 \times 10^{-7}\,\text{m})}$$

$$E \sim 3.5 \times 10^{-19}\,\text{J}$$

We are given that the power of he laser is 100W. A watt is a joule per second. In other words the laser is consuming 100J per second. We previously determined how many joules per photon, so now we can determine how many photons per second.

$$\left(\frac{100\,\text{J}}{\text{s}}\right)\left(\frac{1\ \text{photon}}{3.5 \times 10^{-19}\,\text{J}}\right) \sim 3 \times 10^{20}\ \text{photon/s}$$

6. D

The Bohr radius (0.529 Å) is the most probable distance between the nucleus and the electron in a hydrogen atom in its ground state. It represents the most likely measured value, though values across a range are possible.

7. A

The photoelectric effect is the emission of electrons when light hits a material. Experiments by Einstein involving the photoelectric effect were instrumental in demonstrating the particle (photon) nature of light. Emission of conduction electrons from metals is especially salient, in many cases requiring photon energy of only a few electron-volts, corresponding to short-wavelength visible or ultraviolet light. The photoelectric effect will cause spacecraft exposed to sunlight to develop a positive charge. This can be a major problem, as other parts of the spacecraft are in shadow. The imbalance can discharge through delicate electrical components.

8. C

The transition energy equals the energy of the photon emitted. We are given the wavelength of the emitted photons in angstroms (Å = 10^{-10} m). Before we can compute the photon energy (Planck's constant times the frequency), we will need to convert the wavelength to meters (6564Å = 6.564×10^{-7}m).

$$E = hf$$

$$E = \frac{hc}{\lambda}$$

$$E = \frac{(6.63 \times 10^{-34}\,\mathrm{J{\cdot}s})(3 \times 10^{8}\,\mathrm{m/s})}{(6.56 \times 10^{-7}\,\mathrm{m})}$$

$$E \sim (1 \times 10^{-27})(3 \times 10^{8})\,\mathrm{J}$$

$$E \sim 3 \times 10^{-19}\,\mathrm{J}$$

9. C

The concept that matter behaves like a wave was proposed by Louis de Broglie. The de Broglie wavelength is the wavelength, λ, associated with a particle with mass. It is related to its momentum, $p = mv$, through the Planck constant, h:

$$\lambda = \frac{h}{mv}$$

The Davisson–Germer experiment confirmed the de Broglie hypothesis that matter has wave-like behavior.

The initial intention of the Davisson and Germer experiment was to study the surface of nickel. They fired slow moving electrons at a crystalline nickel target. The reflected electron intensity was measured and was determined to have the same diffraction pattern as those predicted by Bragg for X-rays.

10. D

All matter exhibits wave-like behavior. For example, a beam of electrons can be diffracted just like a beam of light. The de Broglie wavelength is the wavelength, λ, associated with a particle with mass. It is related to its momentum, $p = mv$, through the Planck constant, h:

$$\lambda = \frac{h}{mv}$$

Of the choices presented, the α particle possesses the greatest mass, so if they all have the same speed, the α particle has the shortest de Broglie wavelength.

11. B

Far ultraviolet light is shorter wavelength (122nm - 200nm) than mid UV (200nm - 300nm). Shorter wavelength entails higher frequency, and higher frequency means greater photon energy.

$$E = hf$$

The greater the photon energy, the greater the maximum kinetic energy (and average kinetic energy) of emitted photoelectrons from the metal, so the greater the speed. The maximum kinetic energy equals incident photon energy minus the work function for the metal.

$$K_{mzx} = hf - \phi$$

12. D

If we apply a negative potential to the collector plate and gradually increase it, the photoelectric current decreases, becoming zero at a certain negative potential. This is called the stopping potential. For a given frequency of incident radiation, the stopping potential is determined by the maximum kinetic energy of the photoelectrons that are emitted. The maximum kinetic energy equals incident photon energy minus the work function for the metal.

$$K_{mzx} = hf - \phi$$

For a given frequency of incident radiation, the stopping potential is independent of its intensity.

13. A

In the photoelectric effect experiment electrons are dislodged only when the impinging light reaches or exceeds a threshold frequency. Below that threshold, no electrons are emitted from the material, regardless of the light intensity or the length of time of exposure to the light.

The maximum kinetic energy equals incident photon energy minus the work function for the metal.

$$K_{mzx} = hf - \phi$$

The threshold frequency, f_0, represents a photon energy just enough to overcome the work function, ϕ.

$$\phi = hf_0$$

Photon energy at the threshold frequency equals the work function. This is the minimum energy to liberate an electron from the metal.

14. A

The question stem gives us a stopping potential of -3.50V. At the stopping potential, the electric field between the plates is strong enough to bring even the most energetic photoelecrons to rest before striking the far plate. 3.50 V performs 3.50 eV of work on a single electron. If 3.50 V is the stopping potential, 3.50 eV is the maximum kinetic energy of the photoelectrons.

15. A

As described in the passage, the maximum kinetic energy of the photoelectrons ejected in a trial is given by

$$K_{mzx} = hf - \phi$$

The term ϕ is the work function, ie. the minimum energy required to remove an electron from the surface of the metal.

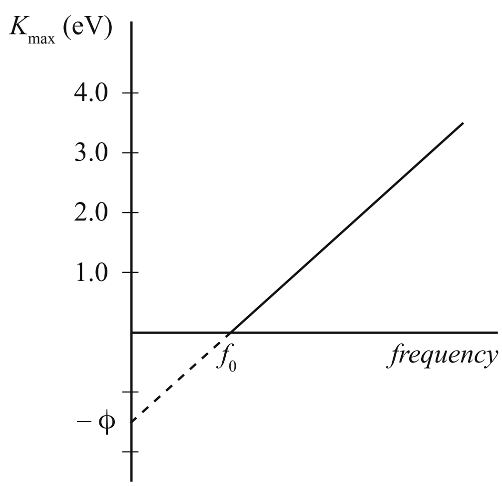
As can be seen in the equation above (and in the graph accompanying the question) the maximum kinetic energy varies linearly with the frequency of the incident radiation. This makes sense because the greater the frequency, the greater the photon energy, so the greater will be the energy possessed by the photoelectron after extraction from the metal.

A common motif in MCAT passages is to turn a question on the association of a linear equation presented in the passage with a graph, where the interpretation of the graph and equation in the light of slope intercept form can yield the values of certain physical quantities.

$$K_{mzx} = hf - \phi$$

$$y = mx + \mathrm{b}$$

On our graph the vertical intercept equals $-\phi$. Extension of the line shows that ϕ is approximately 1.5 eV.

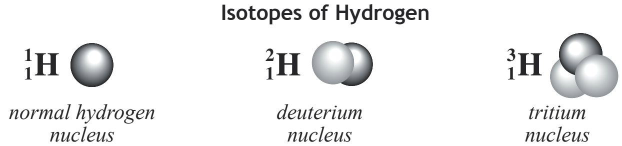


Nuclear Physics
Answers and Explanations

1. D

Isotopes are forms of the same element that contain equal numbers of protons but different numbers of neutrons in their nuclei.

Isotopes of Hydrogen



2. D

Of the answer choices the only uncharged particles are neutrinos and neutrons. However, it is only through rare and special circumstances that the interactions of neutrinos with normal matter may be observed. Chadwick's experiment is credited as being the discovery of the neutron.

3. A

By definition one amu equals 1⁄12 mass of a ^{12}C atom. The rest mass of a free proton is 1.0073 amu, and the rest mass of a free neutron is 1.0086 amu. This may seem contradictory given that there are six protons and six neutrons in the ^{12}C nucleus. How can there be less mass in the ^{12}C atom than there is in twelve free protons and neutrons. The binding energy of carbon-12 is the answer. This is around 92MeV which accounts for the missing mass energy equivalent of 0.064 amu in the difference between twelve free protons and neutrons versus the ^{12}C nucleus.

4. C

The rate of decay in disintegrations per second is called the activity of a sample. The activity depends on the number of nuclei present and their tendency to undergo decay, reflected by the decay constant λ of the radioactive species. Activity is directly proportional to the amount of nuclide present.

$$A = \frac{\Delta N}{\Delta t} = -\lambda N$$

A = activity (disintegrations per second)
N = number of radioactive nuclei
t = time
λ = decay constant

5. A

The SI unit of activity is called the becquerel (Bq):

$$1\text{Bq} = 1 \text{ decay/s}$$

The curie (Ci) (or, more commonly, milli- and micro- curies) is often used instead. The curie was originally calibrated as the activity of one gram of radium.

$$1\text{Ci} = 3.7 \times 10^{10} \text{ decay/s}$$

The rutherford (Rd) is defined as an activity of one million decays per second. It is therefore equivalent to one megabecquerel.

The sievert (Sv) is the SI unit of ionizing radiation dose. It is a measure of the health effect of low levels of ionizing radiation on the human body. The sievert is important in dosimetry and radiation protection. One sievert carries with it a 5.5% chance of eventually developing cancer (based on a linear no-threshold model). One sievert equals 100 rem.

6. B

As described, Cherenkov radiation is emitted when a charged particle passes through a dielectric medium at a speed greater than the phase velocity of light in that medium. The index of refraction of a medium is defined as the ratio of the speed of light in a vacuum to the speed of light in the medium.

$$n = \frac{c}{v}$$

To determine the speed of light in a particular medium, divide the speed of light by the index of refraction.

$$v = \frac{c}{n}$$

$$v = \frac{c}{1.33} = \frac{c}{4/3} = \frac{3}{4}c$$

7. D

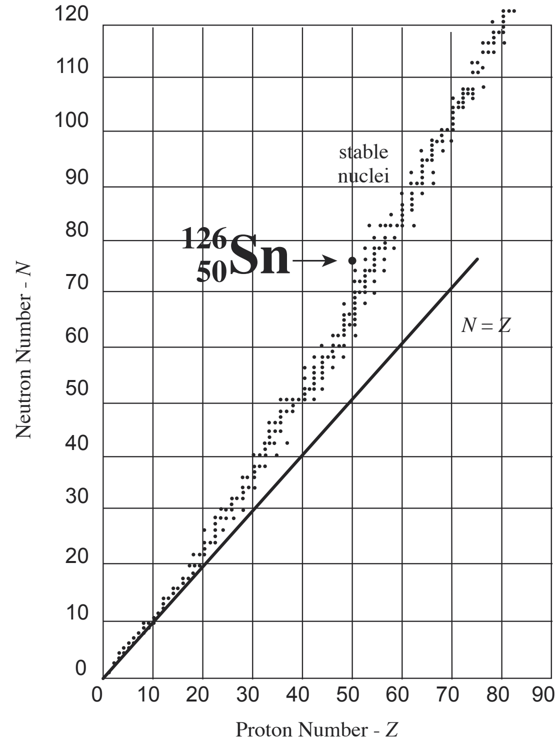
A nuclei undergoes alpha decay by emitting an α particle, which is identical to a helium nucleus – ${}^{4}_{2}\mathbf{He}^{2+}$– two protons and two neutrons. Atomic number, *Z*, decreases by 2 and mass number, *A*, decreases by 4.

8. D

${}^{3}H$, ${}^{14}C$, ${}^{32}P$, and ${}^{35}S$ are β^- emitters. Although ${}^{2}H$, ${}^{13}C$, ${}^{15}N$, and ${}^{18}O$ are also often utilized in life science research, those four are heavy stable isotopes, not radioactive. Unlike the β^- emitters, those isotopes are detected through their effect on the molecular weight of the substance into which they have been substituted.

9. B

The isotope of tin, ${}^{126}_{50}\mathbf{Sn}$, referenced in the question is not within the island of stability. It has too many neutrons. For such radionuclides, β^- emission may occur, leading to a daughter nucleus closer to or within the island of stability. Through β^- emission a neutron in the nucleus will be transformed into a proton.



10. A

In ß⁺ decay, a ß⁺ particle, (a positron, the anti-particle of the electron) and a neutrino, *v*, are emitted. A proton changes into a neutron in the nucleus (Z decreases by 1 with A unchanged).

$${}^{18}_{9}\mathbf{F} \longrightarrow {}^{18}_{8}\mathbf{O} + {}^{0}_{1}\beta^{+} + v$$

11. B

In ß⁻ decay, a ß⁻ particle, which is a high speed electron, and an antineutrino, $\bar{\upsilon}$, are emitted. A neutron changes into a proton in the nucleus (Z increases by 1 with A unchanged).

$${}^{209}_{82}\mathbf{Pb} \longrightarrow {}^{209}_{83}\mathbf{Bi} + {}^{0}_{-1}\beta^{-} + \bar{\upsilon}$$

12. C

As the passage describes, bismuth-213 is obtained after three alpha decays from actinium-225. A nuclei undergoes alpha decay by emitting an α particle, which is identical to a helium nucleus – ${}^{4}_{2}\mathbf{He}^{2+}$ – two protons and two neutrons. Z decreases by

2 and A decreases by 4. The sequence of three alpha decays would begin with actinium-225 then to francium-221 then to astatine-217 and finally to bismuth-213.

13. B

Given that the half-life of B-212 is 60.5 minutes, as the passage indicates, a period of three hours is very close to three half lives. After three half lives, activity will have decreased from 0.32 microCi. to ⅛ value, or 0.04 microCi.

14. D

At the receipt of shipment the sample of actinium-225 has lost more than half of its original activity. In other words, a greater duration of time than a single half-life has passed. The half-life of actinium-225 is given in the passage as 10 days, 12 days is the amount of time which has passed.

15. A

As the passage indicates, they are both beta emitters whose decay reaction may lead to production of a gamma ray. Gamma decay occurs when a nucleus in an excited energy state, very often as the result of a prior decay event, emits a very high energy photon, a gamma ray, as it transitions to a lower energy state. For both of these isotopes, the majority of decay events land on the ground state daughter nucleus, but a percentage with each branches after beta decay to an excited state.

Nuclear Physics Practice Items

1. The species $^1_1\mathbf{H}$, $^2_1\mathbf{H}$, and $^3_1\mathbf{H}$ are

A. allotropes
B. homologs
C. isomers
D. isotopes

2. In 1932 J. Chadwick bombarded beryllium with α-particles and found that highly energetic, uncharged particles were emitted. This neutral radiation could in turn knock protons out of the nuclei of other substances. These particles were

A. positrons
B. neutrinos
C. beta particles
D. neutrons

3. The rest mass of a free proton is

A. greater than 1⁄12 the mass of a ^{12}C atom.
B. less than 1⁄12 the mass of a ^{12}C atom.
C. equal to the rest mass of a free neutron.
D. greater than the rest mass of a free neutron.

4. The decay rate of a radioactive sample is often referred to as its

A. half life
B. rate constant
C. activity
D. emissivity

5. Which of the following is the SI unit for the activity of a quantity of radioactive material?

A. becquerel (Bq)
B. curie (Ci)
C. sievert (Sv)
D. rutherford (Rd)

6. A particle moving through the water surrounding the core of a nuclear reactor emits Cherenkov radiation. Cherenkov radiation is emitted when a charged particle passes through a dielectric medium at a speed greater than the phase velocity of light in that medium. The index of refraction of water is 1.33. Determine the minimum speed at which the particle must be traveling through the water.

A. 0.67 c
B. 0.75 c
C. 1.00 c
D. 1.33 c

7. In α decay

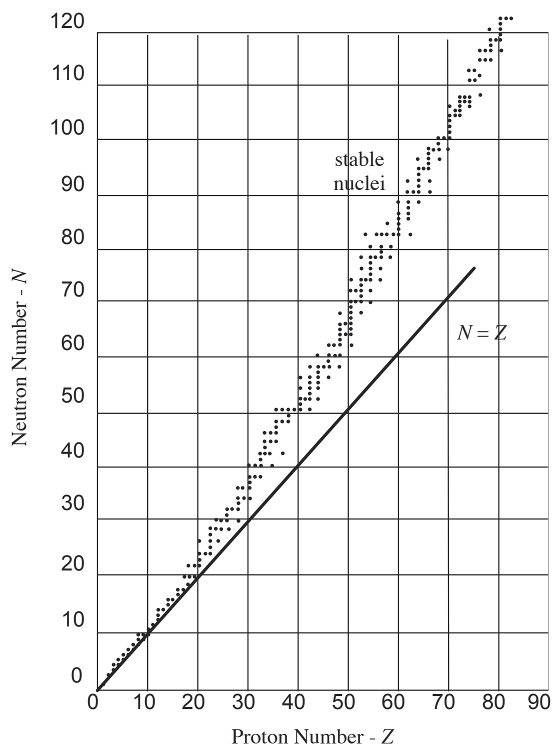
A. Z decreases by 1 and A does not change.
B. Z increases by 1 and A does not change.
C. Z decreases by 4 and A decreases by 2.
D. Z decreases by 2 and A decreases by 4.

8. Which of the following are β^- emitters with a long history of use as radiolabels within life sciences research?

A. ^2H, ^{14}C, ^{18}O, ^{32}P
B. ^2H, ^{13}C, ^{32}P, ^{35}S
C. ^3H, ^{14}C, ^{15}N, ^{35}S
D. ^3H, ^{14}C, ^{32}P, ^{35}S

9. Stable nuclides are represented by a narrow band of proton-to-neutron ratios on the graph of neutron number vs. proton number below. What type of radioactive decay would you expect to occur for the isotope of tin, $^{126}_{50}\mathbf{Sn}$?

A. β^+ decay
B. β^- decay
C. electron capture
D. α decay



10. Which of the following is the daughter nucleus produced by the β^+ decay of fluorine-18?

A. oxygen-18
B. neon-18
C. nitrogen-14
D. fluorine-17

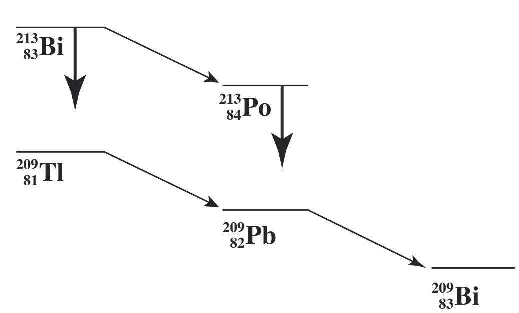
The following passage pertains to questions 11 - 15.

There can be therapeutic benefit to destruction or weakening of cells using radiation. Short-range radiotherapy is known as brachytherapy. Localization in the target organ may occur through the radionuclide being attached to a suitable biological compound. In most cases beta emitters are utilized.

An ideal therapeutic radioisotope is a strong beta emitter with just enough gamma to enable imaging. Lutetium-177 is prepared from ytterbium-176 which is irradiated to become Yb-177 (which decays rapidly to Lu-177). Similarly, yttrium-90 is used for treatment of cancer, particularly non-Hodgkin's lymphoma and liver cancer, and it is being used more widely, including for arthritis treatment. Lu-177 and Y-90 are becoming the main RNT agents.

With a short particle range and high linear energy transfer, α-emitting radionuclides demonstrate high cell-killing efficiencies. Development of therapeutic applications for α-emitting radionuclides is a major research area. For targeted alpha therapy (TAT), actinium-225 is often used, from which the daughter bismuth-213 can also be obtained (via three alpha decays) to label targeting molecules. The bismuth is obtained by elution from an Ac-225/Bi-213 generator in which the Ac-225 is firmly retained by the sorbent, and Bi-213 is eluted with various complexing agents. Bi-213 has a 46-minute half-life. The Ac-225 (half-life 10 days) is formed from radioactive decay of radium-225, the decay product of long-lived thorium-229, which is obtained from decay of uranium-233, which in turn is formed from thorium-232 by neutron capture in a nuclear reactor.

Bi-213 Decay Scheme



Ac-225 itself is an alpha-emitter and may be used directly, bonded to a protein or antibody such as anti-PSMA (anti-prostate-specific membrane antigen) for prostate cancer. Anti-Tac, a monoclonal antibody directed to the human interleukin 2 (IL-2) receptor, has been successfully conjugated to bismuth-212 (half-life 60.5 min) by use of a bifunctional ligand. The alpha decays of Bi-212 and Po-212 are the active ones destroying cancer cells over a couple of hours. Considerable medical research is being conducted worldwide into the use of radionuclides attached to highly specific biological chemicals such as immunoglobulin molecules (monoclonal antibodies). The eventual tagging of these cells with a therapeutic dose of radiation may lead to the regression of some diseases.

11. Which type of radioactive decay is involved in the conversion of $^{209}_{82}\mathbf{Pb}$ into $^{209}_{83}\mathbf{Bi}$?

A. β^+ decay
B. β^- decay
C. electron capture
D. α decay

12. Which of the following is an intermediate in the decay pathway from $^{225}_{89}\mathbf{Ac}$ to $^{213}_{83}\mathbf{Bi}$?

A. $^{209}_{81}\mathbf{Tl}$
B. $^{221}_{88}\mathbf{Ra}$
C. $^{217}_{85}\mathbf{At}$
D. $^{213}_{84}\mathbf{Po}$

13. 0.32 microCi was targeted by Bi-212-labeled anti-Tac to IL-2 receptor-positive adult T-cell leukemia line HUT-102B2. What was the residual activity of the Bi-212-labeled antibodies three hours after administration?

A. 0.02 microCi
B. 0.04 microCi
C. 0.08 microCi
D. 0.16 microCi

14. A sample of Ac-225 possessed an activity of 7.0 mCi at the time of shipment from Oak Ridge National Laboratory. Upon receipt at UCLA its activity was 2.9mCi. How much time elapsed during shipment?

A. 2 days
B. 4 days
C. 8 days
D. 12 days

15. Which of the following is a factor that the radionuclides Lu-177 and Y-90 have in common?

A. Both Lu-177 and Y-90 may decay to a daughter nucleus in an excited state.
B. Compared to α emitters, Lu-177 and Y-90 emit particles which are less penetrating.
C. They are both positron emitters producing gamma photons through pair annihilation.
D. In their decay process, a proton changes to a neutron in the nucleus.

Nuclear Physics
Answers and Explanations

1. D

Isotopes are forms of the same element that contain equal numbers of protons but different numbers of neutrons in their nuclei.

Isotopes of Hydrogen

2. D

Of the answer choices the only uncharged particles are neutrinos and neutrons. However, it is only through rare and special circumstances that the interactions of neutrinos with normal matter may be observed. Chadwick's experiment is credited as being the discovery of the neutron.

3. A

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4. C

The rate of decay in disintegrations per second is called the activity of a sample. The activity depends on the number of nuclei present and their tendency to undergo decay, reflected by the decay constant λ of the radioactive species. Activity is directly proportional to the amount of nuclide present.

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A = activity (disintegrations per second)
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t = time
λ = decay constant

5. A

The SI unit of activity is called the becquerel (Bq):

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6. B

As described, Cherenkov radiation is emitted when a charged particle passes through a dielectric medium at a speed greater than the phase velocity of light in that medium. The index of refraction of a medium is defined as the ratio of the speed of light in a vacuum to the speed of light in the medium.

$$n = \frac{c}{v}$$

To determine the speed of light in a particular medium, divide the speed of light by the index of refraction.

$$v = \frac{c}{n}$$

$$v = \frac{c}{1.33} = \frac{c}{4/3} = \frac{3}{4}c$$

7. D

A nuclei undergoes alpha decay by emitting an α particle, which is identical to a helium nucleus – $^4_2He^{2+}$– two protons and two neutrons. Atomic number, Z, decreases by 2 and mass number, A, decreases by 4.

8. D

^3H, ^{14}C, ^{32}P, and ^{35}S are β^- emitters. Although ^2H, ^{13}C, ^{15}N, and ^{18}O are also often utilized in life science research, those four are heavy stable isotopes, not radioactive. Unlike the β^- emitters, those isotopes are detected through their effect on the molecular weight of the substance into which they have been substituted.

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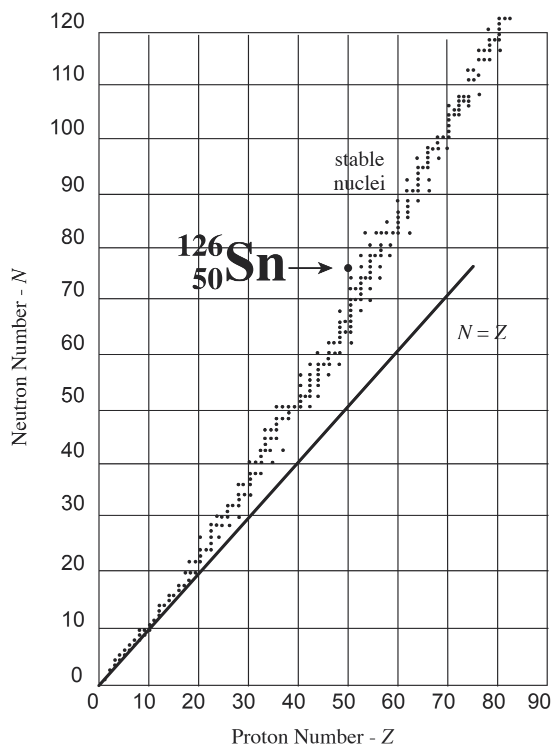

10. A

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$$^{18}_{9}\mathbf{F} \longrightarrow {}^{18}_{8}\mathbf{O} + {}^{0}_{1}\beta^{+} + v$$

11. B

In β^- decay, a β^- particle, which is a high speed electron, and an antineutrino, $\bar{\upsilon}$, are emitted. A neutron changes into a proton in the nucleus (Z increases by 1 with A unchanged).

$$^{209}_{82}\mathbf{Pb} \longrightarrow {}^{209}_{83}\mathbf{Bi} + {}^{0}_{-1}\beta^{-} + \bar{\upsilon}$$

12. C

The passage shows bismuth-213 obtained after three α decays from actinium-225. A nuclei undergoes α decay by emitting an α particle, which is identical to a helium nucleus – $^4_2\mathbf{He}^{2+}$– two protons and two neutrons. Z decreases by 2 and A decreases by 4. The sequence of three alpha decays would begin with actinium-225 then to francium-221 then to astatine-217 and finally to bismuth-213.

13. B

Given that the half-life of B-212 is 60.5 minutes, as the passage indicates, a period of three hours is very close to three half lives. After three half lives, activity will have decreased from 0.32 microCi. to ⅛ value, or 0.04 microCi.

14. D

At the receipt of shipment the sample of actinium-225 has lost more than half of its original activity. In other words, a greater duration of time than a single half-life has passed. The half-life of actinium-225 is given in the passage as 10 days, 12 days is the amount of time which has passed.

15. A

As the passage indicates, they are both beta emitters whose decay reaction may lead to production of a gamma ray. Gamma decay occurs when a nucleus in an excited energy state, very often as the result of a prior decay event, emits a very high energy photon, a gamma ray, as it transitions to a lower energy state. For both of these isotopes, the majority of decay events land on the ground state daughter nucleus, but a percentage with each branches after beta decay to an excited state.