Kinematics

Answers and Explanations

1. B

A quantity is either a vector or a scalar. Vectors are quantities that are fully described by both a magnitude and a direction. Displacement, velocity, and acceleration are vector quantities. In contrast, distance is a scalar quantity. Scalars are quantities that are fully described by a magnitude alone. The distance from Atlanta to Dallas is an equal distance as that from Atlanta to Philadelphia. However, to go from Atlanta to Dallas is a different displacement than to go to Philadelphia. Those are different changes in position. Additionally, it's good to note also that while velocity is a vector quantity, speed is a scalar quantity. Speed is the magnitude of the velocity. If we were driving toward Dallas at 70 miles per hour, that would be the same speed if we were driving 70 mph towards Philadelphia. However, because Dallas is towards the West from Atlanta and Philadelphia is towards the Northeast, those would be different velocities.

2. B

Velocity is the rate of change of the displacement. On a graph the rate of change is the slope of the curve. The change in x divided by the change in t. The graph of displacement with a constant positive velocity is a curve with a constant positive slope, or, in other words, a straight line sloping upwards.

3. B

In the kinematics of one dimensional motion, ie. motion on a line, we use the arithmetic positive and negative to signify direction. Positive means upward vs. negative means downward, or positive means rightward vs. negative means leftward. You can do this because adding and subtracting vectors on a line is just simple arithmetic. However, this sense of a kinematic quantity possibly being negative only applies to vector quantities. Velocity has direction. It does not apply to speed. Speed is the magnitude of the velocity. Whether a body is moving to the right to the left does not signify with regard to its speed, which being a magnitude, can only be positive.

4. C

The acceleration due to gravity is 9.8 m/s^2 directed downward. Just use 10 m/s^2 for the MCAT. For a question like this, you don't want to have to go to a formula. Just think about what 10 m/s^2 means. The velocity is changing ten meters per second per second. Every second the velocity of the ball will change by ten meters per second. For a ball thrown upwards you ask yourself, how long will it take the acceleration due to gravity to deplete the initial upward velocity to zero? If we started upwards at 10 m/s and gravity is going to change that by 10 m/s every second, it's going to reach the peak in 1s.

5. A

With this problem we should treat each 10 s interval separately. For the first 10 s the bus is traveling at a constant velocity, so the displacement is simply the product of this constant velocity and the time. A rate of change times a duration of time equals an amount of change:

$$x - x_0 = v\,\Delta t$$

$$200\text{ m} = (200\text{ m/s})(10\text{ s})$$

For the second 10 s interval, the bus accelerates uniformly to 30 m/s. The displacement equals the product of the average velocity and the time. Because the acceleration is uniform, the average velocity is simply the arithmetic mean of the initial and final velocities:

$$x - x_0 = \frac{1}{2}(v + v_0)t$$

$$250\text{ m} = \frac{1}{2}(20\text{ m/s} + 30\text{ m/s})(10\text{ s})$$

The sum of the displacements of the two intervals is the total displacement: 450 m.

6. D

If constant acceleration had been specified, we would know the average velocity to be the arithmetic mean of the initial and final velocities. However, maybe the dragster accelerated to the final velocity after one second and so was moving at a higher velocity throughout the interval, or it maybe it wasn't until the very end that it accelerated and so it moved with a relatively small velocity for most of the interval. If we knew the net displacement, however, we could determine the average velocity for the interval, but we weren't given the net displacement. In summary, there is not enough information here to determine average velocity.

7. B

Always take a moment to allow a graph to clear. Read the axes, legends and captions. Mentally picture what's happening as if it were in front of you. At 5.0 s the blood transitions from a positive velocity to a negative velocity. It had been moving forward and now it is moving backward.

8. C

The acceleration is the slope of the velocity versus time curve. This occurs twice on the curve, at 2.2 s and 7.2 s. At 7.2 s the negative velocity of the blood had been increasing. It's negative velocity begins to decrease here.

9. A

On the velocity vs. time curve, the blood in the catheter demonstrates a fairly complex pattern motion. It moves forward, speeding up. Then it begins to slow down and reverses course for a period of time, moving backwards. The acceleration is the slope of this curve, and you can see it is constantly changing. Even though the motion is complex, the answer to question of the average acceleration is not hard to determine. The change in the velocity equals the product of the average acceleration and the time interval, or, in other words, average acceleration equals the change in velocity divided by the time:

$$\bar{a} = \frac{v - v_0}{\Delta t}$$

Remember to convert cm/s to m/s! Many MCAT questions are actually tests of focus and attention. Be careful to convert to S.I. units.

$$v - v_0 = 1 \text{ cm/s} = 0.01 \text{ m/s}$$

$$\Delta t = 10 \text{ s}$$

$$\bar{a} = 0.001 \text{ m/s}^2$$

10. D

The velocity is the slope of the displacement vs. time curve. At t_1 the particle is moving with a positive velocity. At the peak of the curve the particle has zero velocity. It comes to rest and begins moving back towards its original position with a negative velocity.

11. B

Crucially, we are told that acceleration is constant. We're given an initial and final velocity and a total displacement. We know that the displacement equals the product of the average velocity and the time, and we know that when acceleration is constant, the average velocity is the simple arithmetic mean of the initial and final velocities. This gives us an easy path to determine the time interval, and if we know the time interval as well as the change in velocity, we can determined the acceleration.

$$x - x_0 = \tfrac{1}{2}(v + v_0)t$$

$$t = \frac{x - x_0}{\frac{1}{2}(v + v_0)} = \frac{400 \text{ m}}{\frac{1}{2}\,(30\text{m/s} + 50 \text{ m/s})} = 10 \text{ s}$$

$$a = \frac{\Delta v}{\Delta t} = \frac{(50\text{m/s - } 30 \text{ m/s})}{10 \text{ s}} = 2 \text{ m/s}^2$$

Alternatively, we could have used the following equation to find the answer more directly, but, perhaps, less intuitively:

$$v^2 = v_0^2 + 2a(x - x_0)$$

12. C

The acceleration due to gravity is -10 m/s^2. In plain English, gravitation changes the velocity 10 meters per second each second. After ten seconds, that will mean a change in the velocity of 100 meters per second. For constant acceleration, this idea is just a simple variation of the idea that an amount of change equals the rate of change times the duration of time:

$$\Delta v = a\,\Delta t$$

13. B

The area under the curve of a rate of change is the amount of change. In other words, the area under the velocity vs. time curve is the displacement.

14. D

All of the statements are true. An object undergoing uniform circular motion moves with a constant speed in a circular path. Although the speed is constant, the velocity is constantly changing in its direction. The velocity vector is tangent to the circular path while the acceleration vector points towards the center.

15. A

First thing to do is reconcile our units in the SI system. We were given the displacement as 3 cm, so we should convert this to meters. Also, because we can see arithmetic involving scientific notation in our future, we should convert it to meters in scientific notation. These kind of set up steps at the start are a good way in problem solving to do something useful while your unconscious mind accommodates itself to the problem.

$$3\text{ cm} = 0.03\text{ m} = 3 \times 10^{-2}\text{ m}$$

We were given an initial and final velocity and told that acceleration is constant. In that case, we know the average velocity will be simply the arithmetic mean of the initial and final. The product of the average velocity and the time interval equals the displacement.

$$x - x_0 = \tfrac{1}{2}(v + v_0)t$$

$$t = \frac{x - x_0}{\tfrac{1}{2}(v + v_0)}$$

$$t = \frac{3 \times 10^{-2}\text{ m}}{\tfrac{1}{2}(6 \times 10^{6}\text{ m/s} + 9 \times 10^{3}\text{ m/s})}$$

Because our initial velocity is almost a thousandth less than our final velocity, it's not going to account for much in the sum of the two, so we can just discard it. As always on the MCAT, numerical answer choices are spaced far apart in their values. The test encourages mental math and simplifying steps.

$$t = \frac{3 \times 10^{-2}\text{ m}}{\tfrac{1}{2}(6 \times 10^{6}\text{ m/s} + 0)}$$

$$t = \frac{3 \times 10^{-2}\text{ m}}{3 \times 10^{6}\text{ m/s}} = 1 \times 10^{-8}\text{ s}$$

16. B

It's generally a safe assumption that an MCAT problem is easier than it looks. Having internalized a few basic ideas will let this one come clear quickly. Firstly, we know that the acceleration is the rate of change of the velocity. An average rate of change over a duration of time maps onto a specific amount of change. We can determine the change in the velocity by plugging the given time values into the expression for velocity as a function of time. From there it's easy to determine the average rate of change over an interval of three seconds.

$$v(t) = \frac{1}{3}\left[(t-1)^{(t+1)}\right]$$

$$v(1) = \frac{1}{3}\left[(1-1)^{(1+1)}\right] = 0\text{ m/s}$$

$$v(4) = \frac{1}{3}\left[(4-1)^{(4+1)}\right] = \frac{1}{3}\,3^{5} = 81\text{ m/s}$$

$$\bar{a} = \frac{v - v_0}{\Delta t}$$

$$\bar{a} = \frac{81\text{ m/s} - 0\text{ m/s}}{3\text{ s}} = 27\text{ m/s}^2$$

17. A

If you know both the initial speed and the angle at which it was launched you know both the magnitude and direction of the initial velocity. The only force on the projectile during flight is gravity which acts downward. Therefore, the vertical component of this vector will determine the time interval for the projectile to reach the peak. How much time will it take gravitational acceleration to deplete the initial vertical velocity to zero?

$$t_{\text{peak}} = \frac{v_{y_0}}{g}$$

The time to land will be twice this value.

$$t_{\text{range}} = \frac{2v_{y_0}}{g}$$

While it's in flight, the horizontal component of the velocity keeps plugging away at a constant value. The horizontal velocity takes the time interval afforded by the vertical velocity and moves the projectile down the field.

$$\text{range} = \frac{2v_{y_0}}{g}\,v_{x_0}$$

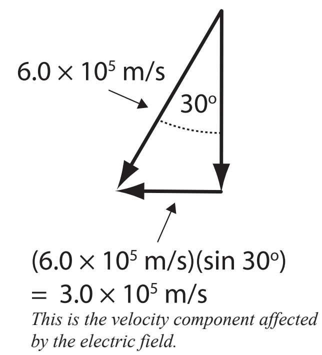
In other words, as long as there is no air friction and the ground is level, the range is completely determined by the initial velocity.

18. A

The negative charge experiences a uniform acceleration perpendicular to the plates towards the positive plate. The motion is directly analogous to projectile motionmotion, so the path will be a parabola.

19. D

Just as with projectile motion, we need to resolve the velocity of the electron to separate out the component which will affected by the acceleration, here due to the uniform electric field instead of gravitation.



As with projectile motion, to answer the question of time in flight, we first need to ask how long it takes the electron to reach the peak. The time in flight will be twice that value.

$$t_{\text{peak}} = \frac{3.0 \times 10^5\text{ m/s}}{1.5 \times 10^{12}\text{ m/s}^2}$$

$$= 2.0 \times 10^{-7}\text{ s}$$

$$t_{\text{range}} = 2\,t_{\text{peak}} = 4.0 \times 10^{-7}\text{ s}$$

20. A

Solving this depends on the previous question, something you'll almost never see on the MCAT, so the question stem provides the time in flight, given as 4.0×10^{-7} s. The question stem asks for the minimum plate separation to prevent the particle from striking the far plate. This is another way of asking what is the distance from the positive plate of the peak of the electron's path. We use half that value, the time to reach the peak, 2.0×10^{-7} s, in the displacement equation. Note acceleration as negative because its direction opposes the initial velocity component.

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

$$x - x_0 = (3.0 \times 10^5\text{ m/s})(2.0 \times 10^{-7}\text{ s}) + \tfrac{1}{2}(-1.5 \times 10^{-12}\text{ m/s}^2)(2.0 \times 10^{-7}\text{ s})^2$$

$$= 3.0 \times 10^{-2}\text{ m} = 3.0\text{ cm}$$