Work & Energy
Answers and Explanations

1. D

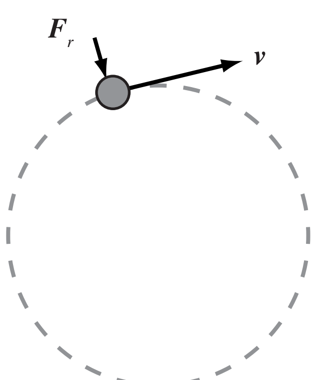
Anything that is moving has kinetic energy. The amount of kinetic energy in a moving body depends directly on its mass and speed.

$$K = \tfrac{1}{2}mv^2$$

Kinetic energy is the work invested in the motion of the body. It also equals the work required to bring it to rest.

2. A

The work performed equals the product of the magnitude of the force component parallel to the displacement and the magnitude of the displacement. In other words, the force needs to at least have a component parallel to the direction the motion to perform work. In uniform circular motion, however, the centripetal force is always perpendicular to the instantaneous displacement. The centripetal force performs no work.



3. D

Firstly, we need to determine the speed of the brick. The acceleration due to gravity is 10 m/s^2 (10 meters per second per second), so after 2s its speed will be 20 m/s.

$$\Delta v = a\,\Delta t$$

$$20 \text{ m/s} = (10 \text{ m/s}^2)(2 \text{ s})$$

Now that we know the speed, we can compute the kinetic energy.

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(5 \text{ kg})(20 \text{ m/s})^2 = 1000 \text{ J}$$

4. C

The work performed equals the product of the magnitude of the force component parallel to the displacement and the magnitude of the displacement, or, as long as you remember that only the component of the force parallel to the direction of the motion is performing work, then it's okay to say that the 'work equals force times distance.'

Here the force required to lift the automobile is the weight of the car ($W = mg$), so the work done is the product of the weight of the car and the height lifted.

$$W = (1000 \text{ kg})(10 \text{ m/s}^2)(10 \text{ m}) = 100{,}000 \text{ J}$$

Note that this is also the potential energy the car now possesses. Work is the vehicle for the transformation of energy.

5. D

The amount of jet-fuel consumed will be proportional to the amount of work done by the engines. The work-energy theorem states that the total work done on an object equals the change in its kinetic energy.

$$W = K_f - K_i = \Delta K$$

The kinetic energy of the jet is proportional the square of its speed.

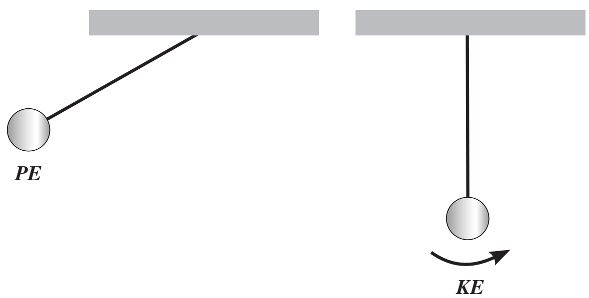
$$K = \tfrac{1}{2}mv^2$$

Because the speed is squared, the same magnitude of change in speed will mean a greater change in kinetic energy at higher speed. For example, the difference between 1 and 3 is the same as the difference between 5 and 7. The difference is 2 in both cases. However, the difference in squares are $3^2 - 1^2 = 8$ versus $7^2 - 5^2 = 24$.

6. D

The recoil of the gun demonstrates Newton's 3rd Law. The floating iceberg illustrates force equilibrium. The example of sliding friction demonstrates dissipation of mechanical energy into thermal energy.

The frictionless pendulum is a model system for the illustration of the conservation of mechanical energy.

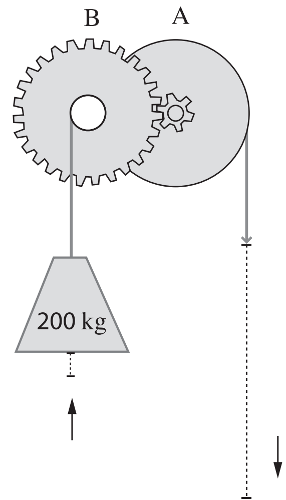


7. B

The purpose of a simple machine is to change the magnitude or direction of an applied force. The central concept in understanding machines is that the work output cannot exceed the work input. For a frictionless system, they are equal: $(\text{force}_{out})(\text{distance}_{out}) = (\text{force}_{in})(\text{distance}_{in})$. The mechanical advantage of a simple machine reflects its force multiplying effect, the ratio of the output force to the input force, by changing the distribution of force and distance constituting the work. In a frictionless system, where no work energy is lost to dissipation, the mechanical advantage also equals the ratio of input distance to output distance. The pulley system has a ratio of input distance to output distance of 2:1. In this pulley system, the input force travels two times the distance in performing the same work as the output force, so it can be two times smaller. The input force is multiplied. To lift the weight, a force only one half as great needs to be input.

8. A

The mechanical advantage of a simple machine reflects its force multiplying effect, the ratio of the output force to the input force, by changing the distribution of force and distance constituting the work. frictionless system, they are equal: $(\text{force}_{out})(\text{distance}_{out}) = (\text{force}_{in})(\text{distance}_{in})$. In other words, to determine the mechanical advantage, the ratio of force_{out} to force_{in}, we need to determine the ratio of the distances.



Let's feel our way with conversion factors. We know that one tooth of gear B moves forward after contacting one tooth of gear A, so we can use this to determine the ratio of revolutions of gear B to gear A.

$$\frac{1}{6}\,\frac{\text{revolution A}}{\text{tooth}} \quad \text{and} \quad \frac{1}{30}\,\frac{\text{revolution B}}{\text{tooth}}$$

$$\left(\frac{6}{1}\,\frac{\text{tooth}}{\text{revolution A}}\right)\left(\frac{1}{30}\,\frac{\text{revolution B}}{\text{tooth}}\right) = \frac{1\text{ revolution B}}{5\text{ revolution A}}$$

The belt advances a circumference per revolution for each gear. We use this to determine the distance ratio.

$$\left(\frac{1\text{ revolution B}}{5\text{ revolution A}}\right)\left(\frac{\text{revolution A}}{(2\pi)(20\text{cm})}\right)\left(\frac{(2\pi)(5\text{cm})}{\text{revolution B}}\right) = \frac{1}{20}$$

$(\text{force}_{out})(\text{distance}_{out}) = (\text{force}_{in})(\text{distance}_{in})$ so the mechanical advantage of this gear train is 20. The weight of 200kg is 2000N (*mg*). Lifting this weight using our gear train, therefore, requires an input force of 100N.

9. D

Like gravitational force, the electrostatic force is conservative. The change in potential energy doesn't depend on the path between two points.

10. B

Kinetic friction forces definitely do perform work. For an object sliding to rest on a surface, the work the kinetic friction force performs represents a transduction of mechanical energy into dissipative forms such as sound and thermal energy. For example, the product of the kinetic friction force for an automobile entering a sliding stop times the distance of the skid-marks equals the work performed by kinetic friction in bringing the automobile to rest. This will equal the kinetic energy of the car before the driver slammed on the brakes.

11. A

We solve this problem using the principle of the conservation of mechanical energy. The block's potential energy at the top of the slide will be fully converted into kinetic energy at the bottom.

$$\frac{1}{2}mv^2 = mgh$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2(10\text{m/s}^2)(1.8 \times 10^{-2}\text{m})}$$

$$v = \sqrt{3.6 \times 10^{-1}\ \text{m}^2/\text{s}^2)}$$

Taking the square root of a number in scientific notation is much easier if the exponent of the base is an even number. It won't change the value of the number if we divide the base portion by 10 and multiply the coefficient by 10 but it makes computing the square root a simple problem.

$$v = \sqrt{36 \times 10^{-2}\ \text{m}^2/\text{s}^2)}$$

$$v = 6 \times 10^{-1}\ \text{m/s}$$

12. D

The electron volt is a unit of energy. An electron volt is the amount of work a 1 volt potential will perform on an elementary charge (the magnitude of charge of a proton or electron). 3000V will perform 300 eV of work on a single electron as it accelerates it within the electron gun of the cathode ray tube. The amount of work the electric field performs within the electron gun will equal the kinetic energy of the electron when it exits, so it exits the aperture of the electron gun with 300 eV of kinetic energy.

$$(3 \times 10^2\ \text{eV})(1.6 \times 10^{-19}\ \text{J/eV}) = 4.8 \times 10^{-17}\ \text{J}$$

$$\frac{1}{2}mv^2 = 4.8 \times 10^{-17}\ \text{J}$$

$$\frac{1}{2}(9.1 \times 10^{-31}\ \text{kg})\ v^2 = 4.8 \times 10^{-17}\ \text{J}$$

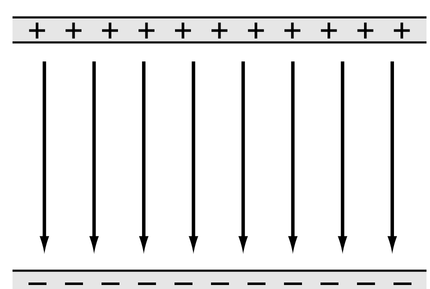
Mental math is a skill the MCAT openly encourages and rewards. Answer choices to quantitative problems are almost always widely spaced. The exam gives you a lot of latitude with mental math. You can see the step of dividing 4.5 into 4.8 as an invitation to simplify the quotient to 1.

$$v \sim \sqrt{1 \times 10^{14}\ \text{m}^2/\text{s}^2}$$

$$v \sim 1 \times 10^{7}\ \text{m/s}$$

13. D

There is a uniform electric field between the plates of a parallel plate capacitor.



Because the particles possess the same magnitude of negative electric charge, the upward force on each particle exerted by the electric field is the same.

$$F = Eq$$

An equal force pushing two particles through the same distance performs the same work (force times distance). This work is the vehicle for the transformation of electrostatic potential energy into kinetic energy.

$$W = K_f - K_i = \Delta K$$

In other words, although Particle B strikes first at a greater speed, each particle strikes the far plate with the same kinetic energy. If two particles possess the same kinetic energy, their speeds will be inversely proportional to the square roots of their masses.

$$\frac{1}{2}m_A\overline{v}_A^2 = \frac{1}{2}m_B\overline{v}_B^2$$

$$\frac{\overline{v}_A}{\overline{v}_B} = \frac{\sqrt{m_B}}{\sqrt{m_A}}$$

A particle which is four times more massive will be moving at half the speed.

14. A

Energy is neither created nor destroyed. The initial potential energy is transformed into a combination of kinetic energy and dissipative forms such as thermal energy and sound through the work of sliding friction.

$$PE_i = KE_f + F_k\Delta x$$

The initial potential energy:

$$PE_i = mgh = (100\text{ kg})(10\text{ m/s}^2)(10\text{ m}) = 10{,}000\text{ J}$$

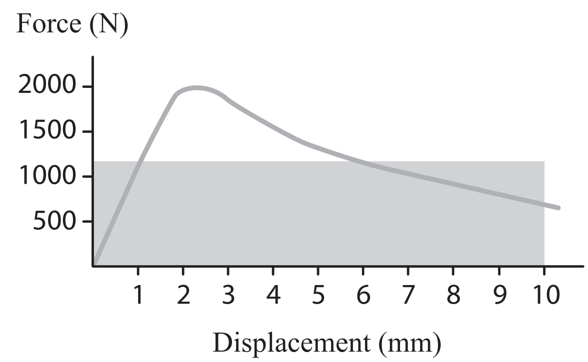
The final kinetic energy:

$$KE_f = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(100\text{ kg})(8\text{ m/s})^2 = 3{,}200\text{ J}$$

Therefore, 6,800 J were lost were lost due to the work of friction. This occurred over 4 s. This represents a rate of work performed, or power, of (6,800 J)/(4 s) = 1,700 W.

15. C

The stored energy equals the work done during compression. This work done equals the area under the force vs. displacement curve. If you find yourself in the position of needing to estimate the area under a curve, draw a best fit rectangle that balances what gets added to what gets left out as well as you can.



Using our rectangle as a guide, the area under the curve is approximately $(1200\text{ N})(1 \times 10^{-2}\text{ m}) = 12\text{ J}$.

16. A

Hydrogen phosphate has -2 total formal charge. ADP possesses a -3 charge. It would require work to move those two molecular ions closer together. When like charges are moved closer together, electrostatic potential energy increases.

17. D

'Work equals force times distance' (simplified when force and displacement are parallel). Fast objects cover greater distance in a given time, so more work is done by a force in that time; i.e. power is greater. The power associated with a force applied to a moving object is the product of the magnitude of the force and the object's speed.

$$P = Fv$$

The locomotive is providing 3000 kW of power to pull rail cars at 36 km/hr. Before using the equation above, we need to convert our units into SI system:

$$P = 3000\text{ kW} = 3 \times 10^6\text{ W}$$

$$v = \left(\frac{36\text{ km}}{\text{hr}}\right)\left(\frac{\text{hr}}{3600\text{ s}}\right)\left(\frac{1000\text{ m}}{\text{km}}\right) = 10\text{ m/s}$$

$$F = \frac{P}{v} = \frac{3 \times 10^6\text{ W}}{10\text{ m/s}} = 3 \times 10^5\text{ N}$$

18. A

Some of the potential energy of the climber below will be transformed into kinetic energy and some will be lost to the work of sliding friction.

$$PE_i = KE_f + F_k\Delta x$$

The initial potential energy:

$$PE_i = mgh = (100\text{ kg})(10\text{ m/s}^2)(15\text{ m}) = 15{,}000\text{ J}$$

To compute the work done by the man's sliding friction above the cliff-edge, we need to determine the kinetic friction force.

$$F_k = \mu_k N = \mu_k mg = (0.8)(100\text{ kg})(10\text{ m/s}^2)$$

$$F_k = 800\text{ N}$$

'Work equals force times distance' (simplified when force and displacement are parallel).

$$Work = (800\text{ N})(15\text{ m}) = 12{,}000\text{ J}$$

Therefore, the climber strikes the ground with 3000J kinetic energy.

$$KE_f = PE_i - F_k\Delta x$$

$$KE_f = 15{,}000\text{ J} - 12{,}000\text{ J} = 3000\text{ J}$$

Determining his speed from the kinetic energy:

$$KE_f = \tfrac{1}{2}mv^2$$

$$\tfrac{1}{2}(100\text{ kg})\,v^2 = 3000\text{ J}$$

$$v = 7.7\text{ m/s}$$

19. D

A monatomic gas molecule such as He possesses only kinetic energy deriving from its linear motion, but diatomic gas molecules such as H_2, O_2, etc. in addition to translational motion, can also rotate and vibrate. With the ability to manifest kinetic energy in both vibrational and rotational modes, a diatomic gas like has more partitions for thermal energy. Equipartition theorem predicts that as a sample of diatomic gas takes in heat, the energy spreads out into all seven of the degrees of freedom shown below. Diatomic gases can absorb heat flow into translational, rotational, and vibrational partitions. For this reason, the molar heat capacity of a diatomic gas will be greater than that of a monatomic gas.