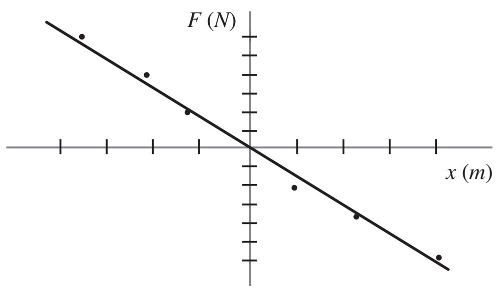
Harmonic Motion Practice Items

1. After mass m is attached to a spring the value of the restoring force F is measured for different values of displacement from equilibrium. The graph below shows the value of the restoring force where x is the displacement of the mass from its equilibrium position.



What is the value of the spring constant k for this spring?

A. -1.5 N/m
B. -0.5 N/m
C. 0.5 N/m
D. 1.5 N/m

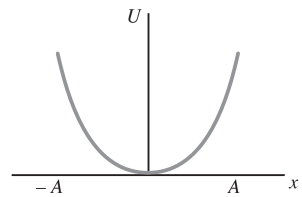
2. The frequency of a simple pendulum on Earth (for small oscillations) depends on the acceleration due to gravity and the length of the pendulum.

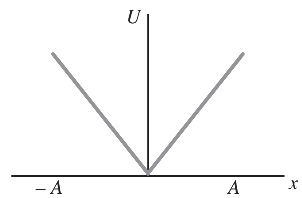
$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

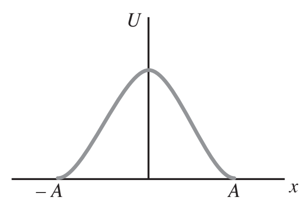
On the surface of the planet Mars, a simple pendulum 43 mm in length oscillates with a period of .65 s. What is the value of the acceleration due to gravity on Mars?

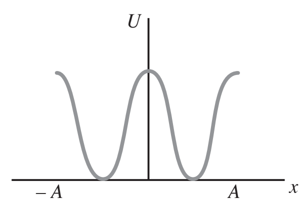
A. 0.67 m/s^2
B. 1.6 m/s^2
C. 2.7 m/s^2
D. 3.8 m/s^2

3. Which of the following graphs shows the elastic potential energy U as a function of displacement x for a simple mass-spring oscillator in the region between the amplitude positions, $x = \pm A$.

A.


B.


C.


D.


4. A particular mass-spring oscillator is not driven or damped. Which of the following would lead to a doubling of the frequency?

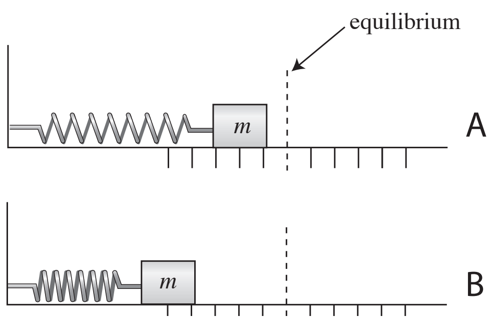
A. halving the mass on the spring
B. quadrupling the strength of the spring
C. doubling the strength of the spring
D. more than one of the above is true

5. Over the course of its oscillations the velocity of a mass-spring will be greatest when

A. the acceleration is greatest
B. the restoring force is zero
C. the displacement equals the amplitude
D. the restoring force is greatest

6. For the two identical springs, A and B, pictured below, displacement from equilibrium is measured as the distance the center of mass m has been moved from its equilibrium position. As can be seen, the two springs have different degrees of compression. The ratio between the two springs of stored potential energy equals

A. 1 : 16
B. 4 : 25
C. 2 : 5
D. 14 : 22



7. A frictionless mass-spring system undergoes simple harmonic motion with an amplitude of 10.0 cm. When the mass is 5.0 cm from its equilibrium position

A. 25% of the oscillator's energy is potential energy and 75% is kinetic energy
B. 50% of the oscillator's energy is potential energy and 50% is kinetic energy
C. 75% of the oscillator's energy is potential energy and 25% is kinetic energy
D. all of the oscillator's energy is potential energy

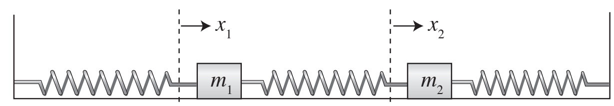
8. Which of the following changes would lead to an increase in the frequency of a simple, frictionless pendulum?

A. increasing the mass
B. decreasing the mass
C. increasing the length
D. decreasing the length

9. What strength of spring should be employed if the goal is to create a frictionless, horizontal mass spring-system that will oscillate a 100g mass at the same frequency as a simple pendulum that is 10 cm in length?

A. 0.1 N/m
B. 1.0 N/m
C. 10 N/m
D. 100 N/m

10. Two identical masses are free to slide over a frictionless horizontal surface. The masses are attached to one another, and to two immovable walls, by means of three identical light horizontal springs of spring constant k, as shown in the figure below. The extensions of the left, middle, and right springs are x_1, $x_2 - x_1$, and $-x_2$, respectively, assuming that $x_1 = x_2 = 0$ corresponds to the equilibrium configuration in which the springs are all unextended.



The acceleration of mass m_1 equals

A. $\dfrac{-kx_1 + k(x_2 - x_1)}{m_1}$

B. $\dfrac{-k(x_2 - x_1) - kx_2}{m_1}$

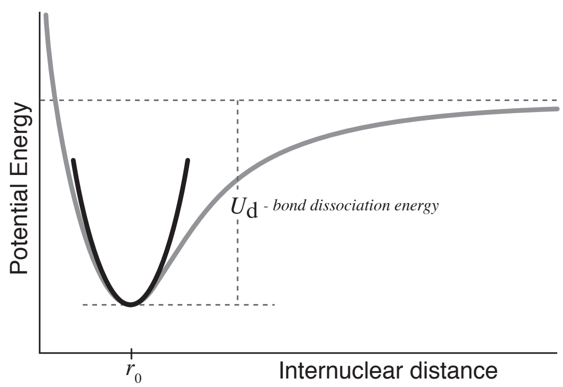
C. $\dfrac{kx_1 - kx_2}{m_1 + m_2}$

D. $\dfrac{kx_1 - kx_2}{(m_1 + m_2)/2}$

Passage (Questions 11-16)

The diatomic harmonic oscillator is a simple model for the vibration of chemical bonds. The model consists of two masses, m_1 and m_2, connected by a Hooke's Law spring with force constant k.

A chemical bond, if stretched too far, will break. A typical potential energy curve for a chemical bond as a function of r, the separation between the two nuclei in the bond is given in the figure below:



The bond dissociation energy curve pictured above (shown in gray in the figure) is given by a function of the form

$$U = U_d\left(1 - e^{-\alpha(r - r_0)}\right)$$

If the energy of the bond is not too high, then the potential energy curve is well approximated by a harmonic oscillator curve (shown in black in the figure). This curve is given by a simpler harmonic oscillator function

$$U = \frac{1}{2}k(r - r_0)^2$$

The exact quantum mechanical description of this model, obtained by inserting the above expression for the potential energy into the one-dimensional Schroedinger equation, yields an infinite set of allowed energies, with equal spacing of successive energy levels.

$$U_n = (n + \frac{1}{2})hf$$

n is a quantum number with discrete values of 0, 1, 2, etc. Planck's constant is denoted by h, and f is the frequency, which is determined by k and the two masses, present as their reduced mass, m_μ.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_\mu}}$$

$$m_\mu = \frac{m_1 m_2}{m_1 + m_2}$$

The reduced mass is a quantity which allows a two-body problem to be solved as if it were a one-body problem of mass m_μ *with the position of one body with respect to the other as the unknown.*

The harmonic oscillator is a useful model because its expression for the potential energy (as a parabola) is an adequate approximation to the true potential energy for many vibrations in real molecules, as long as one confines attention to that part of the potential energy surface close to the minimum. Important problems will arise if we probe the parts of this potential energy surface at much higher energies relative to the minimum.

11. According to information included in the passage the energy of a chemical bond in a highly excited state

A. can be usefully described using a mass-spring model
B. could result in bond dissociation
C. can be approximated as a parabola
D. increases as a function of the square of the internuclear distance

12. In the diatomic harmonic oscillator model of chemical bonds as energy level increases

A. vibrational frequency remains the same
B. potential energy asymptotically approaches the bond dissociation energy
C. the effective mass is reduced
D. molecular motion decreases

13. Which of the following could **not** be described with reasonable accuracy using the quantum mechanical model described in the passage?

A. the minimum photon energy to induce transition in vibrational energy in a covalent bond

B. the minimum photon energy required to break a chemical bond

C. the decrease in the frequency of bond vibration accompanying substitution of a heavier isotope of one of the bond atoms

D. the increase in vibrational frequency with bond strength

14. According to the model described in the passage substitution of two deuterium atoms for normal hydrogen atoms in a hydrogen molecule would result in the following change to the covalent bond?

A. an increase in the bond distance

B. no change in the ground state behavior

C. an increase in the strength of the bond

D. a decrease in frequency of the stretching vibration

15. The infrared spectrograph of a particular organic molecule shows one type of C–C single bond absorbing infrared radiation of wavelength 10100nm (990cm^{-1}) and another bond, a C=C double bond, absorbing radiation at 6042nm (1645cm^{-1}). In terms of the diatomic harmonic oscillator model this difference can best be explained by which of the following?

A. the greater comparative vibrational frequency of C=C double bonds

B. the greater reduced mass of the C–C single bond

C. limitations in the predictive capability of the diatomic harmonic oscillator model

D. the greater comparative strength of C=C double bonds

16. For *k* and reduced masses of the size found in real molecules, the separation of adjacent energy levels, *hf*, is larger than thermal energy at standard temperature. Therefore, according to the model described in the passage,

A. at standard temperature, ground state atoms in chemical bonds are not vibrating.

B. the magnitude of the oscillation in the vibrational ground state is only a small fraction of U_d.

C. nearly all of the oscillators are in their ground state at standard temperature.

D. rotational energy levels of the diatomic molecule have higher wavelengths than the vibrational.