Harmonic Motion
Answers and Explanations

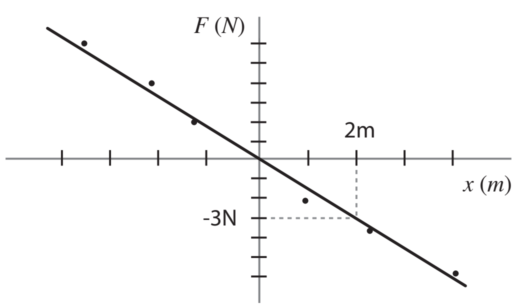
1. B

The restoring force supplied by a spring is proportional to the displacement from equilibrium, how far the spring has been stretched or compressed from the undeformed state. The negative sign in Hooke's Law accounts for the fact that the force exerted by the spring points in a direction opposite to the direction of deformation.

$$F = -kx$$

The proportionality constant, *k*, reflects the strength of the spring. A strong spring has a high *k* and responds to being stretched or compressed a short distance with a high restoring force. The units of *k* are N/m. In other words, *k* tells you how many Newtons of restoring force are supplied by the spring per meter of deformation.

To determine the spring constant for our spring, we can find a convenient point on the best fit line and use those values of displacement and restoring force. The value of 2m for displacement seems like a good place to look.



At 2m displacement, our best fit line predicts a restoring force of -3N.

$$F = -kx$$

$$-3\text{ N} = (-k)(2\text{ m})$$

$$k = 1.5\text{ N/m}$$

2. D

The frequency of a simple pendulum on Mars will depend on the acceleration due to gravity on Mars and the length of the pendulum.

$$f = \frac{1}{2\pi}\sqrt{\frac{g_{mars}}{L}}$$

First, we have a little work to do with the given information.

$$4\text{ cm} = 4 \times 10^{-2}\text{ m}$$

Also, we were given the period of the pendulum. We need to convert that to frequency.

$$f = \frac{1}{T} = \frac{1}{0.65\text{ s}} = 1.5\text{ s}^{-1}$$

Now we can compute g_{mars}.

$$g_{mars} = (2\pi)^2 f^2 L$$

$$= (6.28)^2(1.5\text{ s}^{-1})^2(4.3 \times 10^{-2}\text{ m})$$

$$= 3.8\text{ m/s}^2$$

Because the answer choices are fairly spread out numerically, you have latitude for back of the envelope calculations and mental math for the computation above. You just need to land reasonably close.

3. A

The greater the displacement the greater the restoring force ($F = -kx$). It takes more work to displace the spring a unit increment further from equilibrium. This is why the stored elastic potential energy in a mass-spring goes up with the square of the displacement from equilibrium. The graph is a parabola typical of quadratic functions.

$$U = \frac{1}{2}kx^2$$

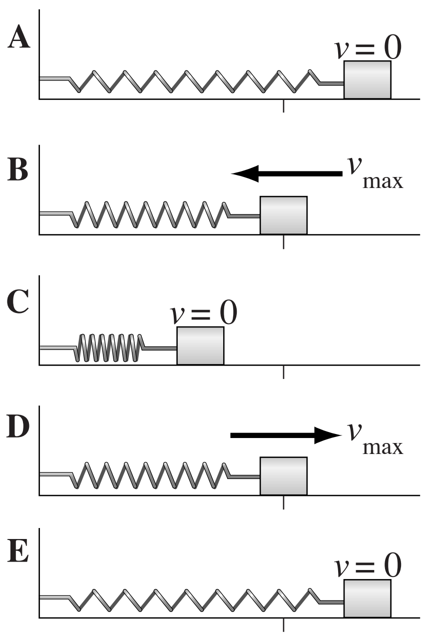
4. B

The frequency of a mass-spring is proportional to the square root of the spring constant.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

5. B

In the snap-shots from a single oscillatory cycle depicted below, the state of the mass-spring depicted in states **B** and **D** correspond to the description in the question stem. As the mass passes the equilibrium position, it is at maximum velocity. All of the energy in the system is in the form of kinetic energy.



6. B

Compression of spring A has displaced the center of its mass 2 units from equilibrium. Compression of spring B has displaced the center of its mass 5 units from equilibrium. The elastic potential energy stored in a spring is proportional to the square of the displacement from equilibrium.

$$U = \frac{1}{2}kx^2$$

Respective displacements of 2 vs. 5 units will correspond to a 4:25 ratio of stored energy.

7. A

The amplitude is the maximum displacement of the mass from its equilibrium position. At that maximum displacement, the mechanical energy of the spring is all the form of elastic potential energy. When moving past the equilibrium position, though, all of the energy is in the form of kinetic energy. For other displacements during oscillation energy apportions as a combination of potential and kinetic energy. Potential energy stored in a spring is proportional to the square of the displacement from equilibrium.

$$U = \frac{1}{2}kx^2$$

Therefore, respective displacements of 5 cm vs. 10 cm will correspond to a 25:100 ratio of stored potential energy. At the 5 cm position, the remaining 75% of total energy will be in the form of kinetic energy.

8. D

The frequency of a simple, frictionless pendulum depends only on its length. Shortening the length of a pendulum increases its frequency.

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

9. C

First we need to convert our units to SI system.

100 g = 0.1 kg
10 cm = 0.1 m

For the frequency of a mass-spring to equal the frequency of the pendulum, the following must be true:

$$\frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

$$\frac{k}{m} = \frac{g}{L}$$

$$k = \frac{mg}{L}$$

$$k = \frac{(0.1\text{ kg})(10\text{ m/s}^2)}{0.1\text{ m}}$$

$$k = 10\text{ N/m}$$

10. A

If you don't know what to do at first with a physics problem at first, try framing the problem as simply as you can. We're asked to find the acceleration of m_1. How do you determine the acceleration of a mass? You determine what forces are acting on it. What forces are being exerted on m_1? The force from the spring on the left and the force from the spring on the right. A spring responds to stretching or compressing by producing a restoring force.

$$F = -kx$$

For a displacement x_1 of the mass m_1 there will be a restoring force $-kx_1$ from the spring on the left. If the second mass m_2 were in a fixed position at $x_2 = 0$, the force from the spring on the right would be simply $-kx_2$, and that would be the net force combined with $-kx_1$. However, the degree of expansion or compression of the spring on the right may be affected by the displacement of m_2, so we need to adjust the force from the right in case the end of the spring is shifted x_2. The combination of the two forces can be expressed and resolved to show the acceleration as below.

$$F = -kx_1 + -k(x_1 - x_2)$$

$$F = -kx_1 + k(x_2 - x_1)$$

$$m_1 a = -kx_1 + k(x_2 - x_1)$$

$$a = \frac{-kx_1 + k(x_2 - x_1)}{m_1}$$

11. B

Like many MCAT passages, the passage combines fundamental concepts from undergraduate general science with concepts that are out of the scope of the expected foreknowledge for the test. Feel your way. Use what you know. Form a provisional understanding. Maintain open questions. How you handle yourself with out of scope material is a figure of merit for the exam.

This particular question combines reading comprehension with a basic sense of the covalent bond. In the passage we are told that the mass-spring approximation can only serve as a model of bond energy at low energies. At high energy, the curve of bond dissociation energy (the gray curve) can be seen approaching a maximum asymptotically. This maximum ($U = 0$; the minimum representing the two nuclei at the bond length is a negative value) represents the bond breaking. With high energy the nuclei can climb out of the well of electrostatic binding energy that is the bond.

12. A

The diatomic oscillator model involves treating the covalent bond as if it were a simple mass-spring system comprised of two masses connected by a spring.

The frequency of a mass-spring system does not depend on the amount of mechanical energy it possesses. Although increasing energy will increase the amplitude of the oscillation, the frequency depends on the spring constant and the mass. For a simple mass-spring, the frequency is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

The same principle applies when there are two masses. As described in the passage, where the formula below is given, the frequency only depends on the spring constant (analogous to bond strength) and the mass (reduced mass in the formula below).

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_\mu}}$$

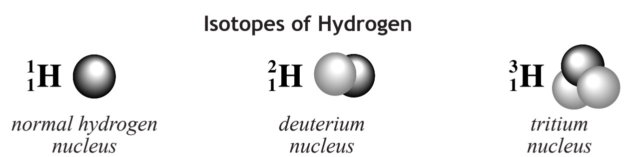
13. B

The quantum mechanical model described in the passage is a quantum mechanical model of the diatomic oscillator (mass-spring). The diatomic oscillator model breaks down at high energy. This can be seen in how the quantum mechanical energy formula below does not approach zero at high n, like the bond dissociation energy curve, but infinity, like the diatomic oscillator curve. It cannot model a bond breaking.

$$U_n = (n + \frac{1}{2})hf$$

14. D

Isotopes of an element have the same number of protons but a different number of neutrons in the nucleus, in other words, the same atomic number, but different neutron number, and, therefore, different mass number. Deuterium is the isotope of hydrogen with mass number 2.



In the diatomic oscillator model the vibrational frequency along the bond axis depends on the spring constant, k (bond strength), and the reduced mass of the two bonded atoms.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_\mu}}$$

$$m_\mu = \frac{m_1 m_2}{m_1 + m_2}$$

Substitution of an isotope would not change the strength of the bond, so k in our model would be unchanged. However, reduced mass, m_μ, does change. Normal hydrogen has a mass of 1 amu, so the two hydrogens together possess reduced mass = ½ amu, while deuterium has a mass of 2 amu, so reduced mass for two bonded deuterium atoms = 1. Increasing reduced mass will decrease the frequency of bond vibration.

15. D

Vibrational frequency along the bond axis depends on the spring constant, k (bond strength), and the reduced mass of the two bonded atoms.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_\mu}}$$

$$m_\mu = \frac{m_1 m_2}{m_1 + m_2}$$

A covalent bond will absorb infrared radiation with frequencies corresponding to its various modes of vibration. The C=C double bond absorbs at a higher frequency (shorter wavelength) than the C–C single bond because the frequency of its stretching vibration is greater. The reduced mass is the same in either case, so the increased frequency of vibration is due to greater bond strength, analogous to spring constant, k, in the diatomic oscillator model.

16. C

Thermal energy is the kinetic energy of the particles, ie. translational, rotational, and vibrational. The quantum mechanical model of the diatomic oscillator described in the passage presents a picture of bond vibration for a single molecule as a series of quantized states of increasing amplitude. Transitioning to a higher energy level in the quantum mechanical diatomic oscillator model requires more thermal energy than is present at standard temperature. That's what the question stem is saying. Only the most vigorous fraction, several standard deviations above the mean kinetic energy, have enough vibrational kinetic energy to do it, so nearly all the molecules are in the ground state.

This question pitches hard, but it's right down the center of the plate. Learn to laugh at questions like this. They are never as hard as they look at first.