Fluid Mechanics
Answers and Explanations

1. C

The specific gravity of a body is the ratio of its density to the density of water. It's convenient to keep our given information in cgs units because we know the density of water in those units.

$$\rho_{H_2O} = \frac{1\text{ g}}{\text{cm}^3}$$

Density equals the mass per unit volume, so the density of our lead sample is:

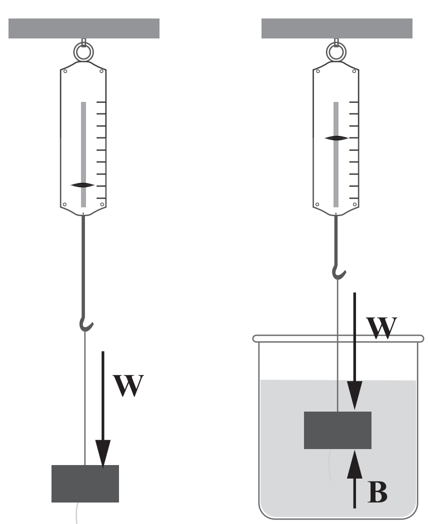
$$V = (2\text{ cm})(3\text{ cm})(5\text{ cm}) = 30\text{cm}^3$$

$$\rho_{Pb} = \frac{330\text{ g}}{30\text{cm}^3} = \frac{11\text{ g}}{\text{cm}^3}$$

Our sample is eleven times more dense than water, so its specific gravity is 11.

2. B

When a physics problem uses the phrase 'the apparent loss of weight' with a submerged object, what is meant by that is the buoyant force.



Archimedes' principle states that the upward buoyant force that is exerted on a body immersed in a fluid, whether fully or partially submerged, is equal to the weight of the fluid that the body displaces.

$$B = W_{\text{fluid displaced}}$$

$$30\text{cm}^3\ H_2O = 30\text{g}\ H_2O$$

$$30\text{g} = .03\text{ kg}$$

$$B = W_{H_2O} = mg = (.03\text{ kg})(10\text{ m/s}^2) = 0.3\text{ N}$$

3. D

The pressure of a liquid open to the atmosphere is equal to the atmospheric pressure on the surface and increases with the depth by an amount equal to the product of the density of the liquid, the acceleration due to gravity, and the depth.

$$P = P_a + \rho g h$$

The atmospheric pressure in SI units is 101,000 Pa or 1×10^5 Pa. The SI density of water is 1000kg/m^3.

$$P = 1 \times 10^5\text{ Pa} + (1000\text{ kg/m}^3)(10\text{ m/s}^2)(40\text{ m})$$

$$P = 5 \times 10^5\text{ Pa}$$

4. D

A high Reynolds' number, rapid interchange of momentum in the fluid, and eddy currents are all characteristics of turbulent flow, not laminar flow, which is steady, streamline flow.

The Reynolds number is a dimensionless quantity empirically found to predict the onset of turbulence. Turbulence is highly probable if the Reynolds number is greater than 2000, and streamline flow is highly probable if the Reynolds number is less than 2000.

$$RN = \frac{\rho v d}{\eta}$$

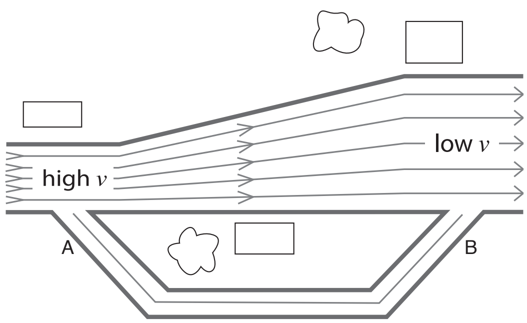
RN = Reynolds number
ρ = fluid density
v = flow speed
d = geometrical property of the flow (diameter of obstruction, pipe width)
η = viscosity

5. B

Because the volume of fluid entering one end of the main canal equals the volume leaving in a given time, the flow can be described by the continuity equation.

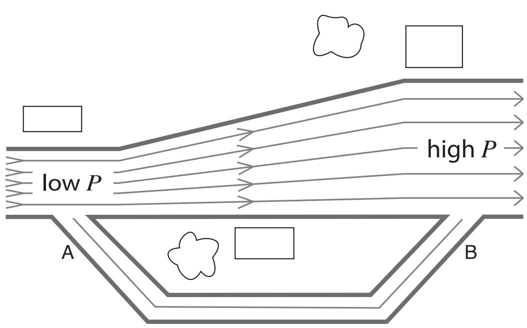
$$A_1 v_1 = A_2 v_2 = \text{constant}$$

As a consequence, the water flowing through the widened section, where the cross-sectional area is greater, is moving more slowly than through the narrow section.



Bernoulli's equation tells us that where the flow speed is slowest, the pressure is greater. (Because the grade is level, $\rho g y$ doesn't change).

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

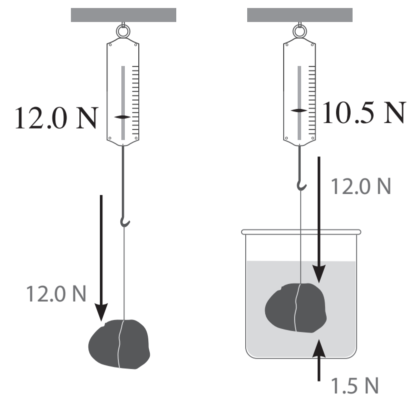


Because the pressure is higher at B than at A and water flows in the "reverse" direction in the secondary canal.

6. B

The spring scale measures a lower weight for the submerged ore because of the upward buoyant force exerted on it. The buoyant force is equal to the weight of the water that the ore displaces.

$$B = W_{\text{fluid displaced}}$$



A 1.5 N apparent loss of weight occurred. This apparent loss of weight is the buoyant force. How can we use this value to determine the specific gravity of the ore, the ratio of the density of the ore to the density of water?

The buoyant force is the weight of the water displaced, measured as an apparent loss of weight of the ore. A submerged object displaces a volume of water equal to its own particular volume. 1.5 N is the weight of this particular volume of water, and this is the same volume as the volume of the ore. Because for a given volume, the weight (mg) of a substance is proportional to its density (m/V), the ratio of the weight of the ore suspended in air to the buoyant force when it is submerged in water must equal its specific gravity. Eureka! The specific gravity is 12.0 N / 1.5 N = 8.0.

7. B

Bernoulli's equation provides a useful framework for understanding the simple siphon.

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

If you compare the profile under Bernoulli's of the static fluid (the flow rate is very low in the reservoir itself) in the upper reservoir at y_{high} to the fluid exiting the pipe into the lower reservoir at y_{low}, there is a transfer of energy per volume element into flow speed driven by the difference in the $\rho g y$ terms. In other words, the atmospheric pressure is not a factor in driving the flow speed. (However, without atmospheric pressure setting a baseline the fluid could not establish a column to get over the high bend of the siphon.)

8. D

Because the volume flux into the siphon has to equal the volume flux exiting the syphon, continuity of volume flux will govern the relationship between the flow speeds.

$$A_1 v_1 = A_2 v_2 = \text{constant}$$

The flow speed is inversely proportional to cross-sectional area. The diameter of tube segment AB is 3 cm. The diameter of tube segment CD is 1 cm. Because the cross-sectional area is πr^2, the ratio of the cross sections of these tube segments is 9:1. The speed through AB will be nine times greater than the speed through CD, or 18 cm/s.

9. D

There is a limit to the height over which one can siphon a fluid. This occurs if the height of fluid in the lower of the two segments is such that the pressure changes with depth, ρgh, would track upwards from a reservoir at atmospheric pressure and predict negative pressure at the height of the siphon. The pressure in the tube cannot be negative. The fluid will come apart and fall down the two columns as air pressure no longer suffices to hold it up in the tube. We can determine the depth of a fluid over which the change in pressure equals atmospheric pressure:

$$\rho gh = 100{,}000 \text{ Pa}$$

$$(1000 \text{ kg/m}^3)(10 \text{ m/s}^2)h = 100{,}000 \text{ Pa}$$

$$h = 10 \text{ m}$$

10. C

Flow depends on the interchange of potential energy per unit volume in the upper reservoir to kinetic energy per unit volume exiting segment CD as described in Bernoulli's law. The sum of these three terms is constant anywhere in the flow line.

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

Because density appears in both of the terms, alterations of density will not affect the flow speed in the lower reservoir. The same logic underlies the independence of mass in governing the speed of a falling body.

However, increasing density will increase Reynolds number. A high Reynolds number is a predictor of turbulent flow.

$$RN = \frac{\rho v d}{\eta}$$

RN = Reynolds number
ρ = fluid density
v = flow speed
d = geometrical property of the flow (diameter of obstruction, pipe width)
η = viscosity

11. C

Through Poiseuille's law we can predict that doubling vessel radius results in an sixteen fold increase in volume flux.

$$Q_{old} = \frac{\Delta P \pi r_{old}^4}{8 \eta l}$$

$$Q_{new} = \frac{\Delta P \pi (2r_{old})^4}{8 \eta l}$$

$$Q_{new} = 16 \frac{\Delta P \pi r_{old}^4}{8 \eta l} = 16\, Q_{old}$$

Volume flux, Q, measured in SI in cubic meters per second, is the rate of volume flow across a unit cross-sectional area in the vessel. Volume flux equals the product of cross-sectional area and flow speed.

$$Q = Av$$

To reiterate, the new volume flux is 16× greater.

$$Q_{\text{new}} = 16Q_{\text{old}}$$

We can express this relationship in terms of cross-sectional area and flow speed.

$$A_{\text{new}}v_{\text{new}} = 16A_{\text{old}}v_{\text{old}}$$

With twice the diameter, the new vessel has four times the cross sectional area.

$$A_{\text{new}} = 4A_{\text{old}}$$

New we can express the new flow speed in terms of the old.

$$4A_{\text{old}}v_{\text{new}} = 16A_{\text{old}}v_{\text{old}}$$

$$v_{\text{new}} = 4v_{\text{old}}$$

12. B

One of the challenges in fluid mechanics is knowing the proper framework to apply. When viscous dissipation is an important factor, such as occurs with flow of a viscous fluid through a narrow vessel, Poiseuille's law applies. The question stem is careful to make the point that the stenosis has not measurably affecting the energy in the flow, so it does not affect the volume flux through the vessel as a whole. In that case, the problem is a straightforward continuity equation problem.

$$A_1v_1 = A_2v_2$$

Decreasing the vessel diameter by 30% will result in a radius 0.7 times the original, so the cross sectional area decreases to 0.49 the original value. The cross-sectional area is approximately half what it was, so the flow speed approximately doubles within the stenosis.

13. D

When blood flow ceases, you have a static fluid. The pressure is the same everywhere at the same depth within a static fluid.

14. A

Turbulent flow is fluid motion characterized by chaotic changes in pressure and flow velocity. It is in contrast to a laminar flow, which occurs when a fluid flows in parallel layers, with no disruption between those layers. Turbulence occurs when kinetic energy in the flow overcomes the damping effect of viscosity. Turbulent flows are always highly irregular. For this reason, turbulence problems are normally treated statistically rather than deterministically. Neither Poiseuille's law nor Bernoulli's equation can be used to model turbulent flow.

A Newtonian fluid refers is one that conforms to the simplest mathematical model of the viscosity (ratio of sheer stress to rate of change of sheer strain). While Bernoulli's equation will deviate the greater the effect of viscous dissipation in the flow of a Newtonian fluid, it is the model system for Poiseuille's law.

15. B

Continuity of volume flux tells us that as vessel diameter increases, flow speed decreases.

$$A_1v_1 = A_2v_2 = \text{constant}$$

Bernoulli's equation tells us that as flow speed decreases pressure increases.

$$P + \tfrac{1}{2}\rho v^2 + \rho gy = \text{constant}$$

16. C

The less the viscosity the less the viscous dissipation of energy along the flow due to peripheral resistance and friction between fluid laminae. You can see this in Poiseuille's law. Imagine viscosity, η, decreasing with a constant volume flux, Q. Do you see how $\Delta P/l$ will decrease? A lower viscosity would lead to a lower loss in pressure per unit length. The fluid is not losing as much energy.

$$Q = \frac{\Delta P \pi r^4}{8 \eta l}$$