Waves

Answers and Explanations

1. D

It's hard to resist answering frequency with this question. It can be this way with the easiest questions on the MCAT. Before toggling the answer, take a moment to check to make sure of what you think you see. It just takes just a moment. The MCAT is always testing your attention.

Frequency is not the wavelength divided by wave speed. That would be the reciprocal of frequency.

$$f = \frac{v}{\lambda}$$

The period is the reciprocal of the frequency. The period tells you how many seconds per cycle.

$$T = \frac{1}{f}$$

2. D

Amplitude is the magnitude of maximum disturbance of the medium, during one cycle of a periodic wave, known as peak deviation. The amplitude of a wave is related to the amount of energy it carries. A high amplitude wave carries a large amount of energy. We talk about the energy of sound in terms of intensity (W/m^2). Intensity goes up with the square of the amplitude of a sound wave.

3. A

The speed of a harmonic wave is the product of wavelength and frequency.

$$v = \lambda f$$

Divide frequency into wave speed to get the wavelength.

$$\lambda = \frac{v}{f} = \frac{343\ \text{m/s}}{660\ \text{s}^{-1}} = 0.5\ \text{m}$$

4. D

Sound waves are longitudinal waves. Vibration is in the same direction as wave propagation.

5. B

Polarization is the phenomenon of transverse waves only. It does not happen in longitudinal waves such as sound waves. The disturbances in longitudinal waves are in the direction of wave propagation. In transverse waves the disturbances are perpendicular to the direction of wave propagation. Electromagnetic waves are transverse and may be polarized.

6. B

The wavelength of the fundamental vibrational mode of a stretched string is twice the length of the string. If the length of the stretched string isn't altered, the wavelength of the fundamental mode doesn't change. However, the speed of the wave on the string does change. The speed is proportional to the square root of the tension.

$$v = \sqrt{\frac{F}{\mu}}$$

Frequency equals wave speed divided by wavelength.

$$f = \frac{v}{\lambda}$$

A three fold increase in wave speed corresponds to a three fold increase in frequency.

7. B

The wavelength of the fundamental mode is twice the length of the string, so the wavelength is the same for all of the banjo strings. However, frequency is proportional to the wave speed and the wave speed is different.

$$f = \frac{v}{\lambda}$$

And wave speed is proportional to the square root of the tension and inversely proportional to the mass per unit length.

$$v = \sqrt{\frac{F}{\mu}}$$

In other words, the frequency is inversely proportional to the mass per unit length.

A banjo string is an extended cylinder. The mass per unit length is proportional to the volume per unit length. The volume per unit length equals the cross-sectional area. The cross sectional area is proportional to the square of the diameter.

In summary! The frequency is inversely proportional to the square root of the square of the diameter, or, more plainly, frequency is inversely proportional to the diameter.

8. C

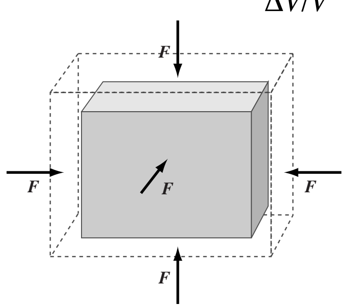
The speed of sound in a particular medium equals the square root of the bulk modulus of the medium divided by its density.

$$v = \sqrt{\frac{B}{\rho}}$$

Sound travels more slowly in air than water, but, by itself, the lower density of air vs. water predicts sound to travel faster in air. The reason sound travels more slowly in air is that air has a much lower bulk modulus.

The bulk modulus is a property of a material. Its the ratio of an applied pressure to the percent compression observed of the material.

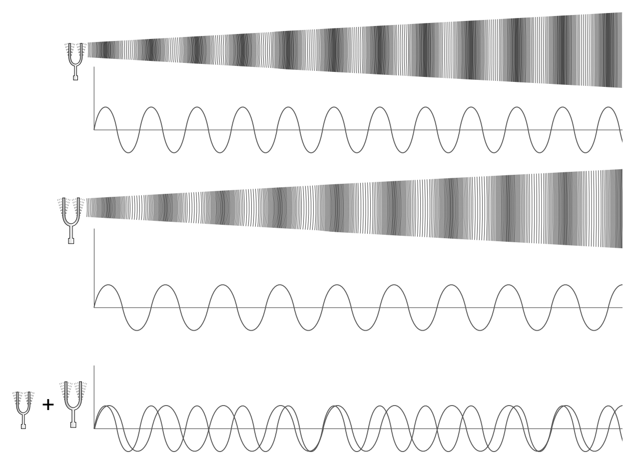
$$\text{bulk modulus} = -\frac{F/A}{\Delta V/V}$$



A high value of B indicates a material resists compression, while a low value indicates volume appreciably decreases under uniform pressure. The reciprocal of the bulk modulus is compressibility, so a substance with a low bulk modulus has high compressibility. Air has a much higher compressibility than water, in other words, a much lower bulk modulus. Hence, sound travels more slowly in air than water despite the lower density of air.

8. C

Beats are fluctuations in sound intensity produced when two tones nearly equal in frequency are sounded simultaneously. When sounds of slightly different frequencies are in the same position in space, the sound waves are periodically in phase, resulting in constructive interference, and then out of phase, resulting in destructive interference. The resulting audible pulsations in intensity are called beats.



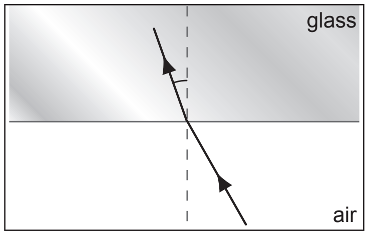
The number of beats one hears per second, the beat frequency, is simply the arithmetic difference between the frequencies of the contributing sound waves.

$$f_b = f_1 - f_2$$

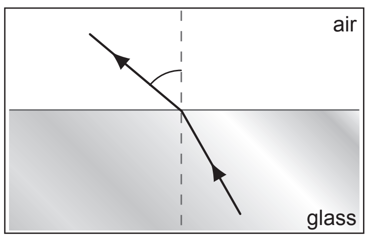
Our tuning fork produces a 9 Hz beat frequency with the 392 Hz tone and a 5 Hz beat frequency with the 406 Hz tone. 9 Hz from 392 and 5 Hz from 406, its frequency must be 401 Hz.

10. A

In the transmission of light, as light moves from a fast to a slow medium it bends towards the normal due to refraction.

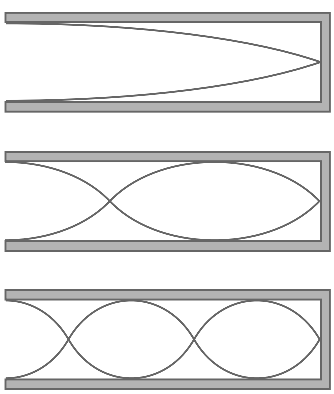


As light is transmitted from a slow to a fast medium it bends away from the normal.



11. A

The wavelength of the fundamental mode of the standing waves in an air column closed at one end is four times the length of the pipe.



$$\lambda_n = 4L, \frac{4L}{3}, \frac{4L}{5}, \ldots \frac{4L}{n}$$

$$(n = 1,3,5,\ldots)$$

Hence, the wavelength of the fundamental mode is

$$\lambda_n = 4L = 4(0.75 \text{ m}) = 3.0 \text{ m}$$

To determine frequency when you have the wavelength, divide the wavelength into the wave speed.

$$f = \frac{v}{\lambda} = \frac{330 \text{ m/s}}{3 \text{ m}} = 110 \text{ Hz}$$

12. A

Intensity measures the actual energy flux produced by a sound source in watts per square meter.

The loudness scale is related to intensity, but it is based on how humans perceive sound, and is proportional to the logarithm of intensity.

$$\beta = 10 \log \left(\frac{I}{I_0}\right)$$

If I_1 is the original intensity, doubling the number of amplifiers will produce a new intensity of $2I_1$. The decibel level with this new intensity will be:

$$\beta_2 = 10 \log \left(\frac{2I_1}{I_0}\right) = 10\left[\log \left(\frac{I_1}{I_0}\right) + \log 2\right]$$

Doubling the intensity results in the addition of 10 log (2) decibels, or about 3.

If you don't remember that the common logarithm of 2 is about 0.3, then ask yourself, to what power do I need to raise 10 to get 2? Well, 2 is a bit less than the cube root of 10, so the logarithm of 2 is a bit less than one third.

A faster path for answering this particular question would be to remember the following:

Increase intensity 10 fold and you add 10 dB.

Increase intensity 100 fold and you add 20 dB.

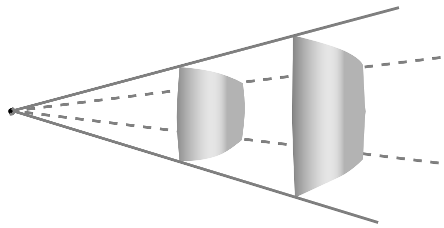
Increase intensity 1000 fold and you add 30 dB.

We only doubled intensity, so the addition to loudness is necessarily less than 10 dB. Choice 'A' is the only possibility.

13. A

Sound waves spread out into space from a point source. The spatial volume into which the sound waves are spreading increases and the intensity decreases with the square of the distance from the point source. The increasing volume of space into which the sound has spread dilutes its intensity.

$$I \propto \frac{1}{r^2}$$



The loudness scale is related to intensity, but it is based on how humans perceive sound, and is proportional to the logarithm of intensity.

$$\beta = 10 \log\left(\frac{I}{I_0}\right)$$

In our problem, the sound level at a distance 5 m from a point source is 100 dB. To determine the distance at which the sound level would be 80 dB remember that a decrease in 20 dB represents a 100 fold decrease in intensity. To *decrease* intensity 100×, we need to *increase* the distance from the point source 10×. The sound level will be 80 dB at 50m.

14. A

The Doppler effect causes the frequency measured by an observer to be different than the frequency of the source whenever there is relative motion between the source and observer. A coefficient can be derived, depending on the speeds of the waves, observer, and source, to adjust the source frequency to the observer frequency.

$$f' = f\left(\frac{v \pm v_0}{v \mp v_s}\right)$$

f' = observed frequency
f = source frequency
v = wave speed
v_o = speed of observer
v_s = speed of source

If the observer is moving toward the source, use the positive sign in the numerator, which would lead to an increased observed frequency. If the observer is moving away from the source, use the negative sign. If the source is moving toward the observer, use the negative sign in the denominator, which would lead to an increased observed frequency. If the observer is moving away from the source, use the negative sign.

In our problem, the observer is stationary. The first equation below describes the shift in observed frequency as the plane approaches the tower, the second after it has passed.

$$440 \text{ Hz} = f\left(\frac{350 \text{ m/s}}{350 \text{ m/s} - v_s}\right)$$

$$330 \text{ Hz} = f\left(\frac{350 \text{ m/s}}{350 \text{ m/s} + v_s}\right)$$

At this stage we can foresee combining these two equations to solve for v_s.

$$440 \text{ Hz}\,(350 \text{ m/s} - v_s) = f\,(350 \text{ m/s})$$

$$330 \text{ Hz}\,(350 \text{ m/s} + v_s) = f\,(350 \text{ m/s})$$

It makes sense to hold off on doing any arithmetic. Combining the equations should simplify things for us.

$$(440 \text{ Hz})(350 \text{ m/s}) - (440 \text{ Hz})\,v_s = f\,(350 \text{ m/s})$$

$$(330 \text{ Hz})(350 \text{ m/s}) + (330 \text{ Hz})\,v_s = f\,(350 \text{ m/s})$$

We subtract the second equation above from the first and then solve for v_s.

$$(110 \text{ Hz})(350 \text{ m/s}) - (770 \text{ Hz})\,v_s = 0$$

$$350 \text{ m/s} - 7v_s = 0$$

$$v_s = 50 \text{ m/s}$$

15. A

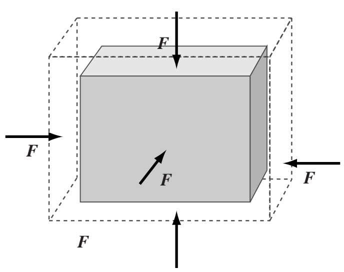
These are periodic but non-sinusoidal wave forms. With sinusoidal waves, the wavelength is the distance from crest to crest or trough to trough, but the true definition of the wavelength is the distance over which the wave's shape repeats. These all have the same wavelength.

16. D

For both P waves and S waves, wave speed is greater with greater elastic modulus. P waves are compression waves. Both shear and bulk modulus are relevant with P waves. S waves are transverse. The only relevant modulus with S waves is shear modulus.

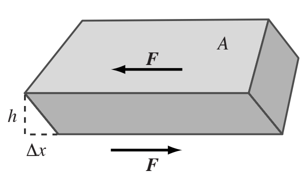
Bulk modulus the ratio of an applied pressure to the percentage compression observed of the material. The higher the bulk modulus the more rigid the material is to pressure.

$$\text{bulk modulus} = -\frac{F/A}{\Delta V/V}$$



The higher the shear modulus the more rigid the material is to a shearing stress.

$$\text{shear modulus} = \frac{F/A}{\Delta x/h}$$



Whether P waves or S waves, the more rigid the medium the faster the wave. The more dense the material the slower the wave. Choice 'D' is the only answer which is logically consistent with this underlying basis for wave speed.

17. A

S waves are transverse. P waves are longitudinal. Only transverse waves can be polarized. With transverse waves, the disturbances are perpendicular to the direction of wave velocity. There can be variation in the plane of oscillation transverse to wave velocity. Polarization describes selection preference in this geometric orientation. With longitudinal waves on the other hand, the oscillations only occur along the wave parallel to wave velocity. It doesn't make sense to speak of "polarization" of longitudinal waves.

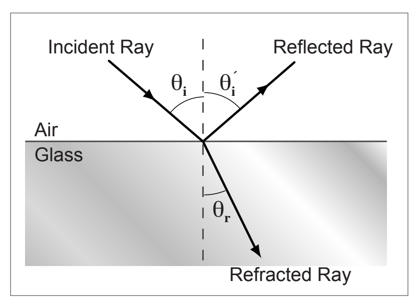
18. A

You can see this in the expressions for wave speed. By mathematical necessity, the expression for P wave speed on the left below must be greater than S wave speed on the right.

$$\sqrt{\frac{\frac{4}{3}B + S}{\rho}} > \sqrt{\frac{S}{\rho}}$$

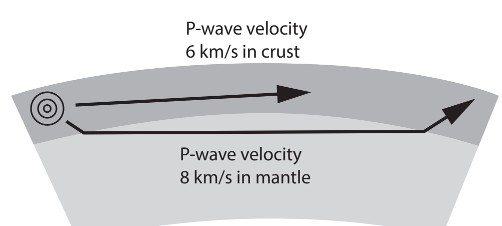
19. B

The figure below shows a similar process of reflection and refraction of electromagnetic waves (light) at the boundary between air and glass.



20. D

The mental projection in reading comprehension should be along the lines of the figure below.



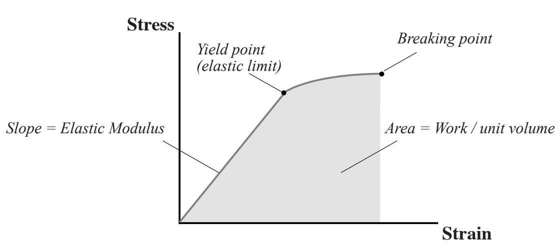
21. B

A teaching purpose with this passage is to provide a bit of an orientation for the topic of elastic deformation. Elasticity isn't mentioned as a specific item on AAMC's MCAT topic outline. However, it intersects with several important topics that are on the outline, for example, what determines the speed of sound in a particular medium. Elasticity is right on the edge of the scope of the test. The speed of sound in a particular medium equals the square root of the bulk modulus of the medium divided by its density. Bulk modulus is an elastic property of the medium which describes how hard the material is to compress.

$$v = \sqrt{\frac{B}{\rho}}$$

The elastic properties of a solid body govern the manner of its deformation under external forces. A stress is the array of forces causing a deformation. Strain measures the degree of deformation. The elastic modulus governs the relationship between stress and strain below the elastic limit, also called the yield point. After the yield point has been reached, the deformation is no longer completely reversible (the object no longer resumes its original shape when the stress is removed). If the elastic modulus for a particular material has a high value, objects made from that material are difficult to deform (and a reversible deformation stores much energy for a given strain).

$$\text{elastic modulus} = \frac{\text{stress}}{\text{strain}}$$



Curve of **Stress vs. Strain** *which describes the mechanical deformation of a solid body.*

Three types of elastic modulus describe the three major kinds of deformation, Young's modulus (tensile stress and strain), shear modulus (shear stress and strain), and bulk modulus (volume stress and strain).

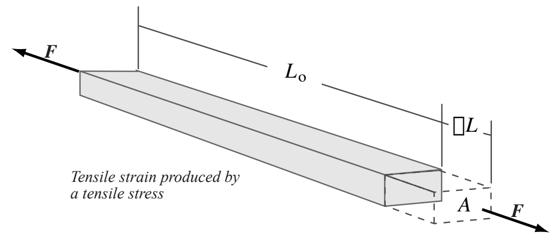
$$\text{Young's modulus} = \frac{F/A}{\Delta L/L_o}$$

tensile stress $= \frac{F}{A}$

F = force
A = cross-sectional area

tensile strain $= \frac{\Delta L}{L_0}$

ΔL = change in length
L_0 = original length



Tensile strain produced by a tensile stress

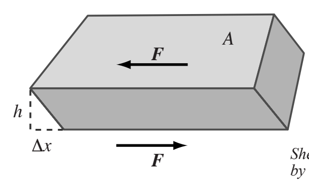
$$\text{shear modulus} = \frac{F/A}{\Delta x/h}$$

shear stress $= \frac{F}{A}$

F = tangential force
A = area of face being sheared

shear strain $= \frac{\Delta x}{h}$

Δx = horizontal distance sheared face moves
h = height of object



Shear strain produced by a shear stress

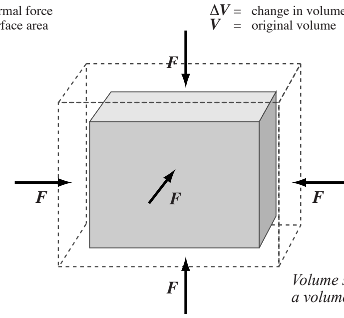
$$\text{bulk modulus} = -\frac{F/A}{\Delta V/V}$$

volume stress $= \frac{F}{A}$

F = normal force
A = surface area

volume strain $= \frac{\Delta V}{V}$

ΔV = change in volume
V = original volume



Volume strain produced by a volume stress

S waves are transverse. Solid geological materials can transmit S waves because a solid body can support a shear stress. In liquids, though, a shear stress simply causes the fluid laminae to slide past each other. A liquid supports a volume stress but not a shear stress. For this reason only P waves, which are longitudinal, can transmit through the liquid core.