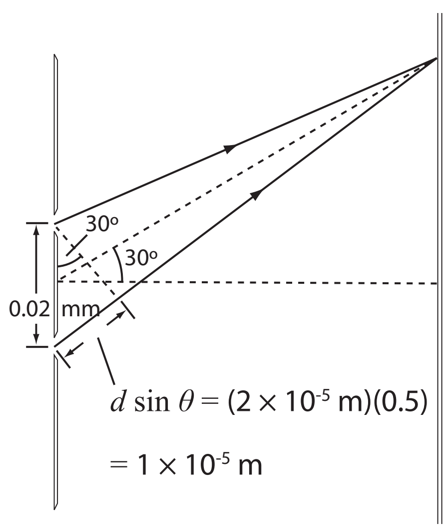
Wave Optics
Answers and Explanations

1. A

Diffraction is the bending of waves around the corners of an obstacle or through an aperture into the region of geometrical shadow of the obstacle. The diffracting object or aperture effectively becomes a new source of the propagating wave. Diffraction can be understood in terms of the Huygens principle that treats each point in a propagating wave-front as a collection of individual spherical wavelets. The characteristic interference pattern resulting from diffraction is most pronounced when light from a coherent, monochromatic source (such as a laser) encounters a slit/aperture that is comparable in size to its wavelength.

2. B

The extra distance from the lower slit is the product of the spacing between the two slits, d, and the sine of the angle, θ, to the point on the screen mentioned in the passage.

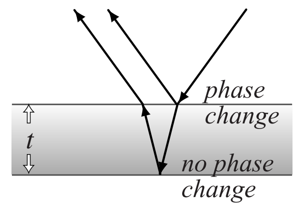


3. B

$|X'Y| - |XY|$ equals the extra distance traveled by the reflected ray. One might think that a bright fringe occurs if this extra distance is an integral number of wavelengths ($n\lambda$), but remember that when light reflects within a fast medium off of a high index of refraction surface it undergoes a 180° phase change, so we need to offset the condition for constructive interference by half a wavelength.

4. A

In thin film interference, light reflected from the front and rear surfaces of a thin film combines to form a resultant wave. Whether constructive or destructive interference occurs depends on whether the combining rays are in or out of phase. The phase difference of the rays depends on the wavelength of light in the film medium, the thickness of the film (assume the light rays are nearly normal to the surface), and whether or not there are any phase changes with either reflection. Hard reflection is reflection off of a medium of higher index of refraction and leads to a 180° change of phase. Soft reflection is reflection off of a medium of lower index of refraction and does not produce a change of phase.



Normally, in interference problems, a path difference (2t with thin films) equal to an integral number of wavelengths produces constructive interference, but if one of the reflections is hard and the other soft the result is destructive interference.

Condition of Constructive Interference (with one reflection having a phase change)

$$2t = (m + \tfrac{1}{2})\lambda_n \quad (m = 0, 1, 2, \ldots)$$

In other words, if the thickness of the soap film is one fourth the wavelength of the incident light, 147nm, the path difference would be half the wavelength. Because one of the reflections resulted in a 180° change of phase, this produces constructive interference.

5. D

Diffraction leads to bending of the light around the coin into the region of its shadow. The edges of the coin effectively becomes a new source of the propagating light. The diffracting rays from different points along the entire edge circumference will have traveled exactly the same distance to the very center of the coin, so they will be in phase there and produce a bright spot resulting from constructive interference.

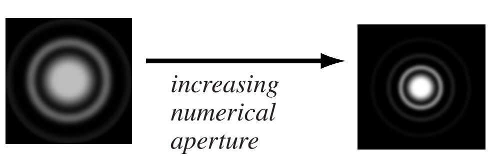
6. D

Because there is no path difference from either slit to the position of the central fringe, all of the variously colored components of white light will be in phase in the center. Elsewhere on the screen, however, the phase difference of the light rays from the two slits will depend on the wavelength, so at a certain angle only a certain wavelength will be in phase while others will be undergoing destructive interference.

7. D

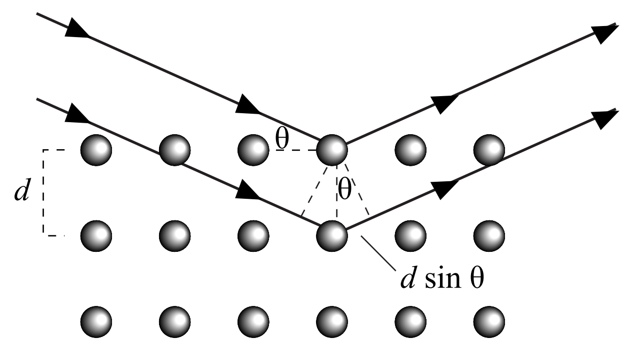
Diffraction of light passing through a circular aperture produces an interference pattern similar to single slit diffraction. By Huygen's principle, each portion of the aperture acts as a source of waves. For a given image point, the interference of wavelets yields a diffraction pattern known as an airy disk. Widening the slit narrows the central maximum in single slit diffraction. Likewise, widening a circular aperture increases the phase difference possible for different light paths, increasing the number of diffraction orders captured, which decreases the size of the central maxima of the airy disk image of a given point source.

When the central maximum of one airy disk falls on the first minimum of another (Rayleigh's criterion), the images are said to be just resolved. If the width of the pupil were greater, the human eye could resolve finer detail. This would increase the number of diffraction orders captured and decrease the size of the central maxima of object point airy disc patterns. Decreasing the size of central maxima increases the ability to resolve two airy disk patterns.



8. B

The basic principles underlying X-ray crystallography are similar to thin film interference. It can be seen in the figure below that the path length difference for light reflecting off of the top layer of the crystal and the adjacent layer below it is $2d \sin \theta$.



Bragg's Law describes how constructive interference will be observed if this path length difference is equal to an integral number of wavelengths of the incident light.

$$2d \sin \theta = n\lambda \qquad (n = 1,2,3 \ldots)$$

The first diffraction order would at an incident angle of 30° if the crystal layer spacing exactly equaled the wavelength of the incident X-rays.

$$2d \sin 30^\circ = (1)\,\lambda$$

$$d = \lambda$$

9. A

In birefringent materials, such as calcite and quartz, the index of refraction is not the same in all directions. Double refraction causes an unpolarized light beam to be split into an ordinary (O) ray and an extraordinary (E) ray, which are polarized in mutually perpendicular directions. The two images seen when the block is placed on the paper are produced by these two rays respectively.

A polaroid film only allows the components of the electric field vibrations to pass that are parallel to its transmission axis. Because the ordinary and extraordinary rays are polarized in mutually perpendicular directions, the rotating polaroid film will allow a varying intensity of transmission of each which is phase shifted in the rotation. In other words, the two images will alternately appear and disappear.

10. C

Specific rotation is common standard for optical rotation. It allows us to compare samples collected under different concentrations and path lengths.

$$[\alpha] = \frac{\alpha_{observed}}{c \times l}$$

observed rotation → $\alpha_{observed}$; specific rotation → $[\alpha]$; concentration g/ml → c; path length dm → l

11. A

Circular dichroism spectroscopy is based on the differential absorption of left-hand and right-hand circular polarized light. Circularly polarized light occurs when the direction of the electric field vector rotates about its propagation direction. At a single point in space, the circularly polarized-vector will trace out a circle over one period of the wave frequency.

The far-UV CD spectrum of a protein can reveal information about the secondary structure of the protein. The technique can be used to estimate the fraction of the protein in the alpha-helix or beta-sheet conformations, for example.

12. B

A basic scenario repeats itself throughout wave optics from Young's interference, thin layer interference, to the Michelson or Mach-Zehnder interferometers. Light which was originally in phase and traveling together is separated in some way, whether by diffracting through separate slits (or different points in the same aperture in single slit diffraction), reflecting off different boundaries in a thin layer, or being split by a beam splitter to travel different paths in interferometry. The rays then recombine. Interference results. The different optical paths they have followed may or may not have produced a phase difference leading to either constructive or destructive interference.

13. B

At detector 2, in the absence of a sample, the sample beams and reference beams will arrive with a phase difference of half a wavelength, yielding complete destructive interference. When light traveling in a fast medium reflects off a boundary to a slow medium, the light undergoes a 180° phase shift. This is known as a hard reflection. The reference beams arriving at detector 2 will have undergone a 180° phase shift of due to one hard reflection. The sample beams arriving at detector 2 will have undergone two hard reflections. Therefore, when there is no sample, only detector 1 receives light.

14. A

An alteration of optical path occurs as the collimated sample beams travel through the sample. The collimated beams are variously altered across the wave front. When they recombine with the reference beam, the phase shift differences create an image of the sample as an interference pattern on both detectors. Because both the sample and reference beams on their paths to detector 1 undergo two hard reflections, while on their paths to detector 2 undergo two and one hard reflections respectively, there is a 180° offset in the phase shift relationships reflected in the images on the two detectors, so they will appear as negative images of each other.

15. C

The collimated beams of white light passing through the glass of the sample cell will have undergone dispersion. Dispersion occurs in glass because the phase velocity of light in glass varies slightly with the frequency of light, ie. the index of refraction in glass is slightly different for the different colors. Without the compensating cell present in the path of the reference beam, the dispersion occurring in the sample cell will cause the optical path of the sample beam to slightly different for each frequency. This will lead to a pattern of colored fringes in each detect

16. C

As the gas is being evacuated from the sample cell, the index of refraction within is steadily decreasing from the value for the gas at its original concentration to the vacuum value of 1. As the index of refraction decreases, the wavelength of the light within the sample cell increases. For light of a given frequency, the faster the medium the lower the index of refraction and the longer the wavelength.

$$\frac{v_2}{v_1} = \frac{n_1}{n_2} \qquad \frac{\lambda_2}{\lambda_1} = \frac{v_2}{v_1} \qquad \frac{\lambda_2}{\lambda_1} = \frac{n_1}{n_2}$$

As the wavelength in the sample cell increases, eight phase shift cycles occur in the interference of the sample and reference beams. In other words, it required eight more wavelengths to cross the sample cell containing the gas than the vacuum. Those eight extra wavelengths are subtracting from the optical path of the sample beam as the sample cell is being evacuated.

A helpful clue in the question stem might make the problem easier to conceptualize and solve. In addition to presenting the wavelength in nm, it also provides the same value in the form of wavenumber, $\tilde{v}$. The wavenumber is the reciprocal of the wavelength. A value of 20,000 cm^{-1} tells you that there are 20,000 cycles per centimeter for this particular frequency of light in a vacuum. The sample cell itself is 1 cm long. The wavenumber of the light in the gas was therefore 20,008 cm^{-1}. Because wavenumber is the reciprocal of the wavelength, which is inversely proportional to the index of refraction, wavenumber is directly proportional to index of refraction.

$$\frac{n_{gas}}{n_{vacuum}} = \frac{2.0008 \times 10^4\ cm^{-1}}{2.0 \times 10^4\ cm^{-1}}$$

$$\frac{n_{gas}}{1} = \frac{2.0 \times 10^4 + 8}{2.0 \times 10^4}$$

$$n_{gas} = 1 + \frac{8}{2.0 \times 10^4}$$

$$n_{gas} = 1 + 4.0 \times 10^{-4} = 1.0004$$